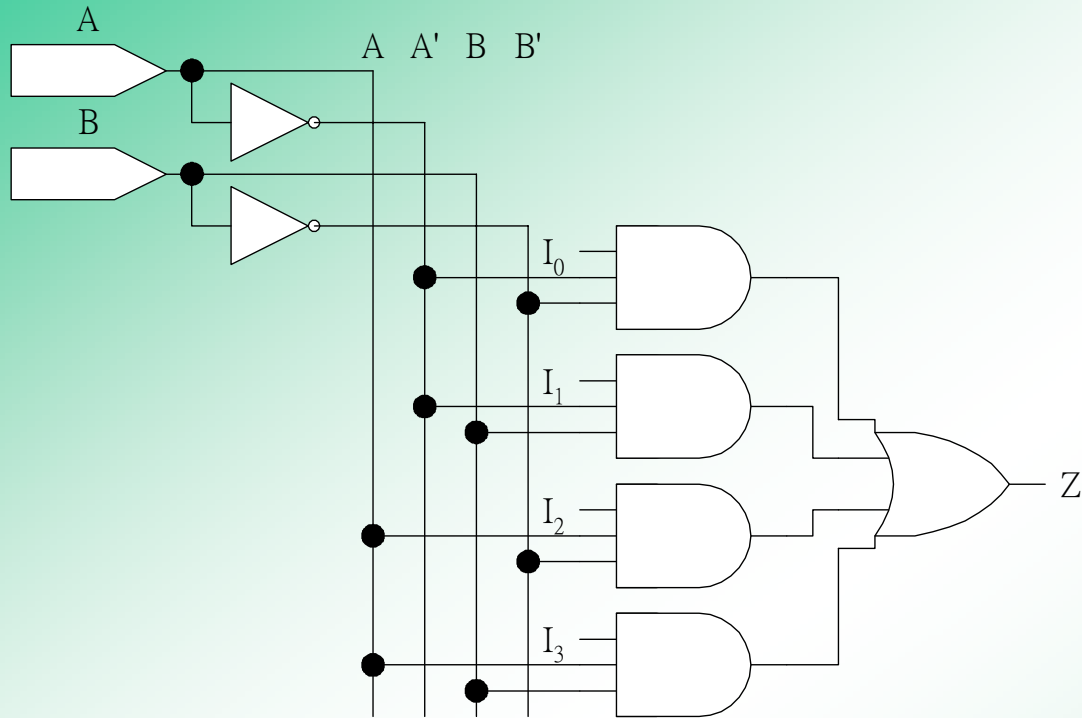


# **Unit 9.**

## **Multiplexers, Decoders, and Programmable Logic Devices**

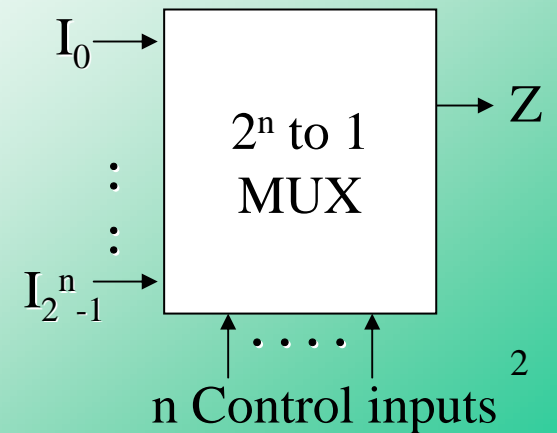
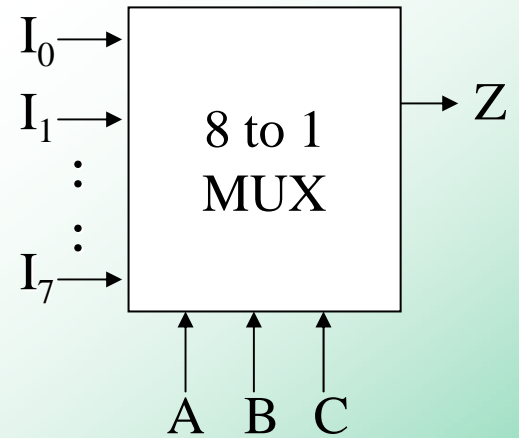
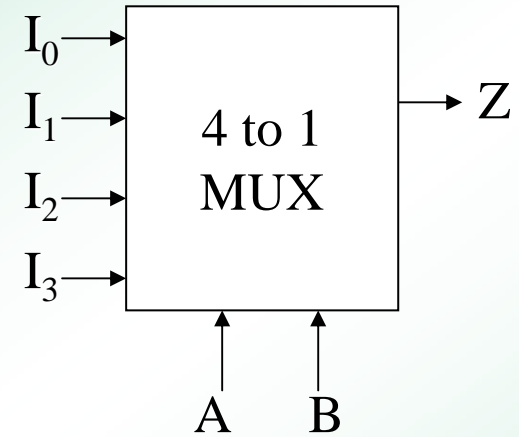
# Multiplexers (Data Selectors)



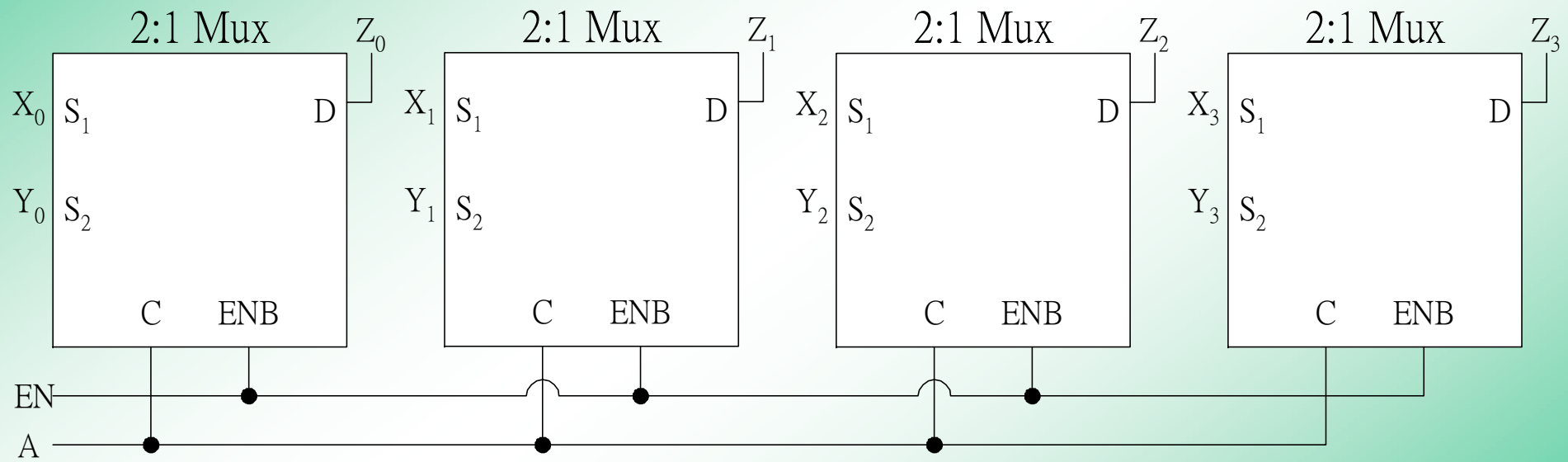
$$4-1: Z = A'B'I_0 + A'BI_1 + AB'I_2 + ABI_3$$

$$8-1: Z = A'B'C'I_0 + A'B'CI_1 + A'BC'I_2 + A'BCI_3 \\ + AB'C'I_4 + AB'CI_5 + ABC'I_6 + ABCI_7$$

$$2^n - 1: Z = \sum_{k=0}^{2^n - 1} m_k I_k$$



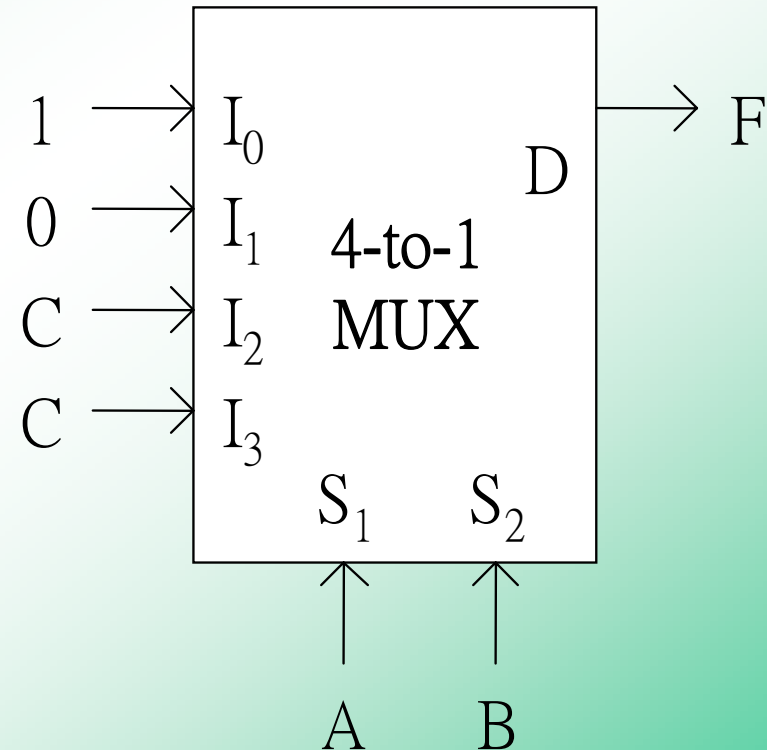
# Quad Multiplexer to Select Data



## 4-to-1 MUX to realize 3-variable function

Ex :

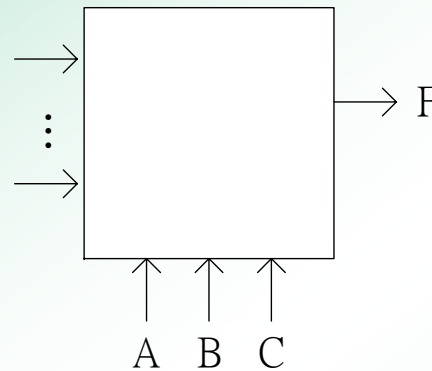
$$\begin{aligned} F(A, B, C) &= A'B' + AC \\ &= A'B' + AC(B + B') \\ &= A'B' \cdot 1 + AB'C + ABC \end{aligned}$$



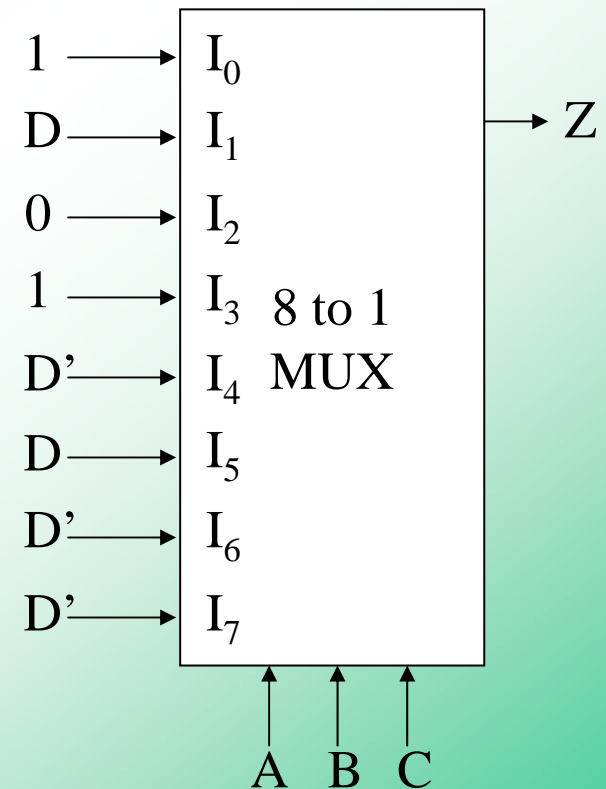
## 8-to-1 MUX to realize 4-variable function

$$F = A'B'C' + A'BC + ABD' + AC'D' + B'CD$$

$$= A'B'C'(D+D') + A'BC(D+D') + AB(C+C')D' + A(B+B')C'D' + (A+A')B'CD$$



$$\begin{aligned} A'B'C' &= 1 \\ A'B'C &= D \\ A'BC' &= 0 \\ A'BC &= 1 \\ AB'C' &= D' \\ AB'C &= D \\ ABC' &= D' \\ ABC &= D' \end{aligned}$$



	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>F</i>	
<i>I</i> <sub>0</sub>	0	0	0	0	1	1
	0	0	0	1	1	
<i>I</i> <sub>1</sub>	0	0	1	0	0	<i>D</i>
	0	0	1	1	1	
<i>I</i> <sub>2</sub>	0	1	0	0	0	0
	0	1	0	1	0	
<i>I</i> <sub>3</sub>	0	1	1	0	1	1
	0	1	1	1	1	
<i>I</i> <sub>4</sub>	1	0	0	0	1	$\overline{D}$
	1	0	0	1	0	
<i>I</i> <sub>5</sub>	1	0	1	0	0	<i>D</i>
	1	0	1	1	1	
<i>I</i> <sub>6</sub>	1	1	0	0	1	$\overline{D}$
	1	1	0	1	0	
<i>I</i> <sub>7</sub>	1	1	1	0	1	$\overline{D}$
	1	1	1	1	0	

$$F = A'B'C'(D+D') + A'BC(D+D') + AB(C+C')D' + A(B+B')C'D' + (A+A')B'CD$$

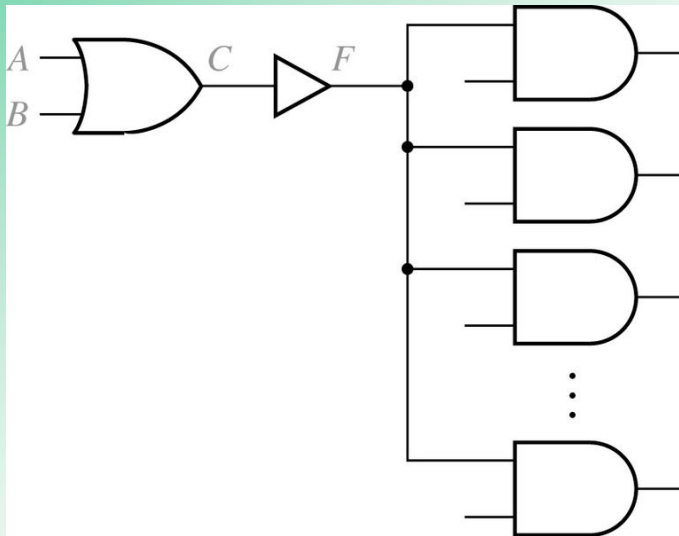
<i>I</i> <sub>0</sub> <i>I</i> <sub>1</sub>	<i>I</i> <sub>2</sub>	...	<i>I</i> <sub><i>n</i>-1</sub>	<i>F</i>			
10	0	...	0	0	0	1	1
11	0	...	1	0	1	0	1
				↓	↓	↓	↓
				0	<i>I</i> <sub><i>n</i></sub>	$\overline{I}_n$	1

$$F = A'B'C' + A'B'CD + A'BC + AB'C'D' + AB'CD + ABC'D' + ABCD$$

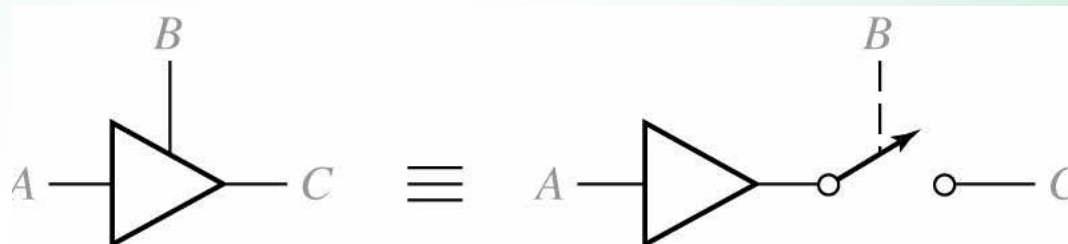
$$= A'B'C' + A'BC + ABD' + AC'D' + B'CD$$

## § Three - state buffers

Buffers: to increase the driving capability of a gate output



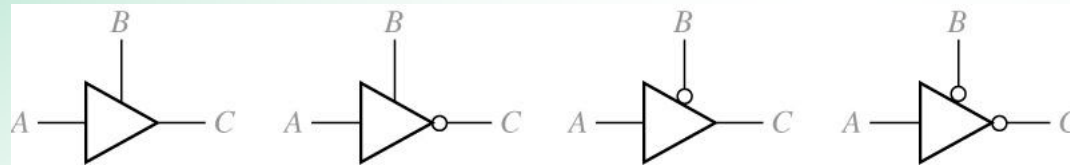
Tri-state Buffers: permits gate outputs to be connected together



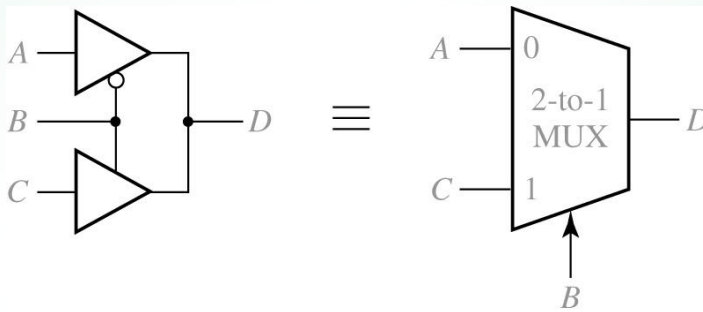
## Truth tables

<i>B</i>	<i>A</i>	<i>C</i>
0	0	Z
0	1	Z
1	0	0
1	1	1

*Truth Table*

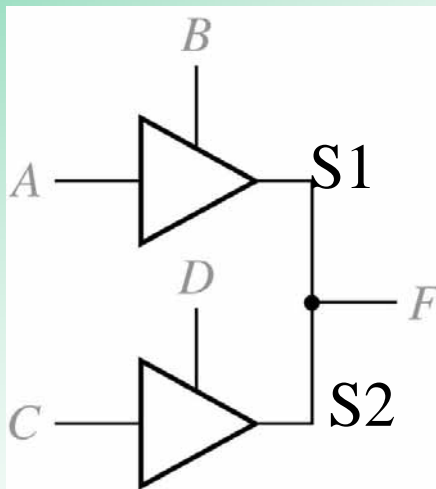


## Data selection using three-state buffers





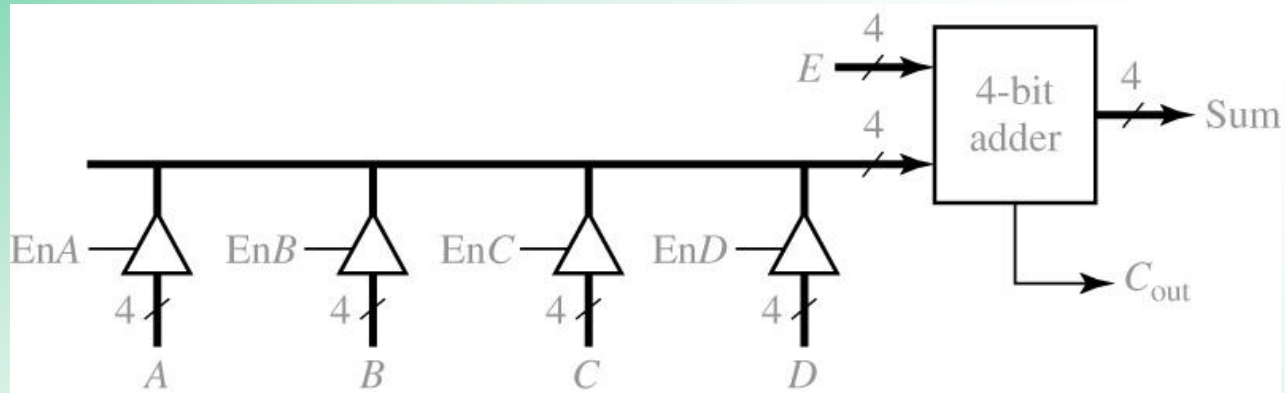
## Circuits with two three state buffers



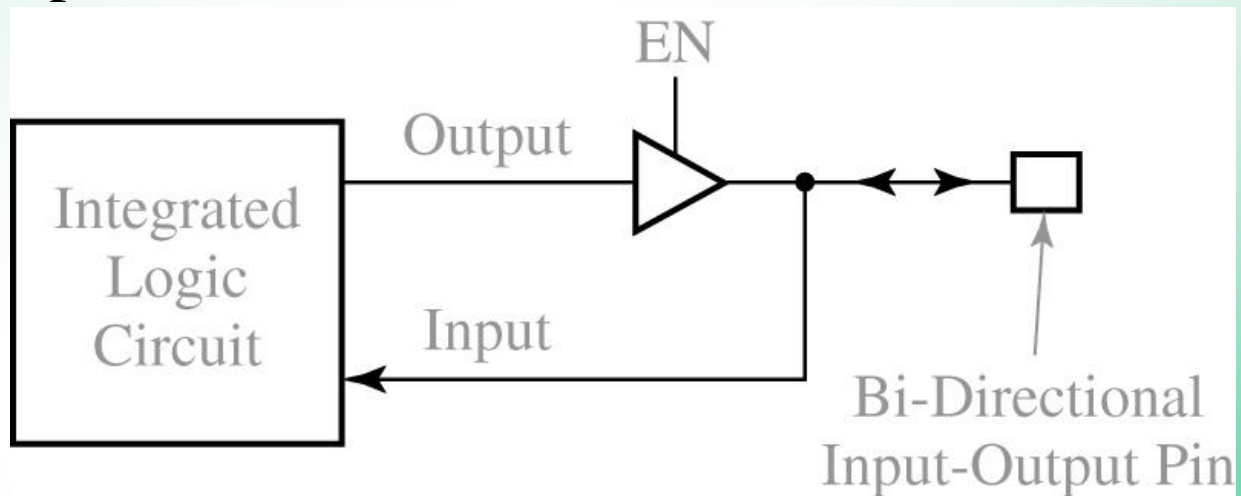
S1	S2			
	X	0	1	Z
X	X	X	X	X
0	X	0	X	0
1	X	X	1	1
Z	X	0	1	Z

# Application of three-state buffer

## 1. Bus

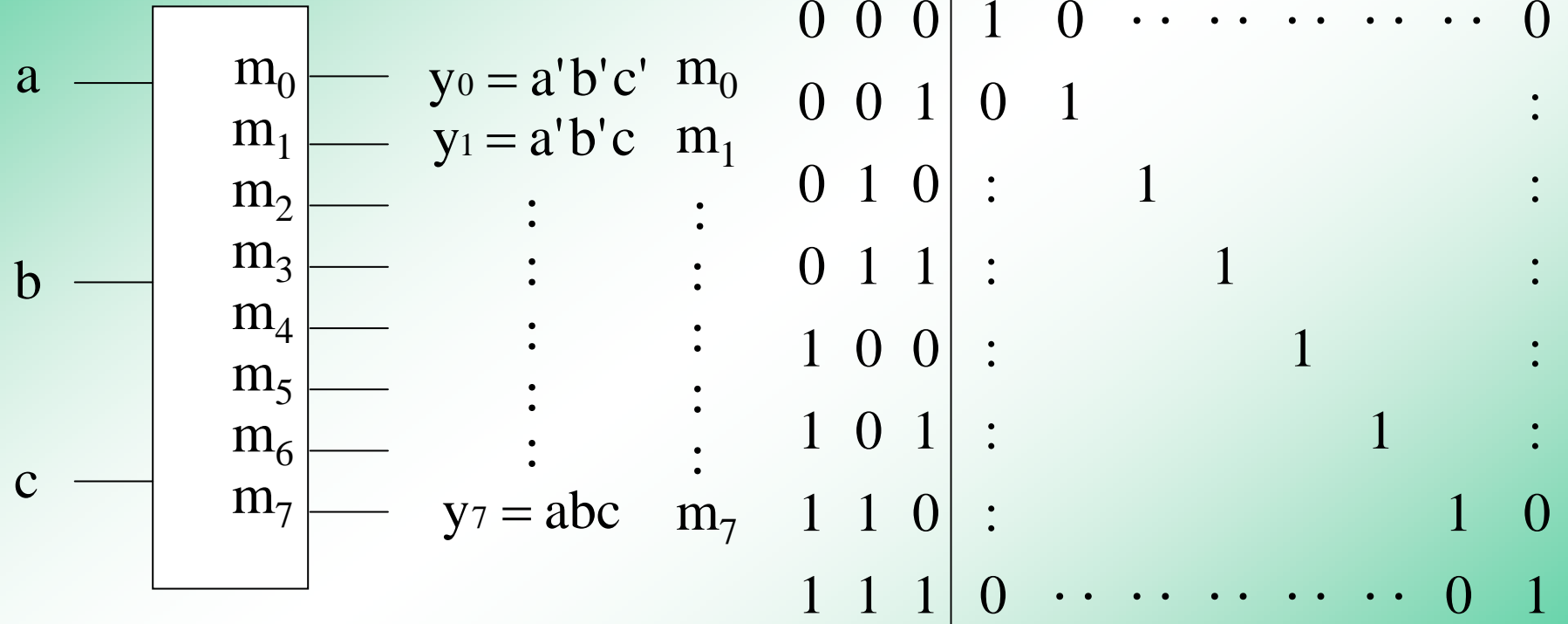


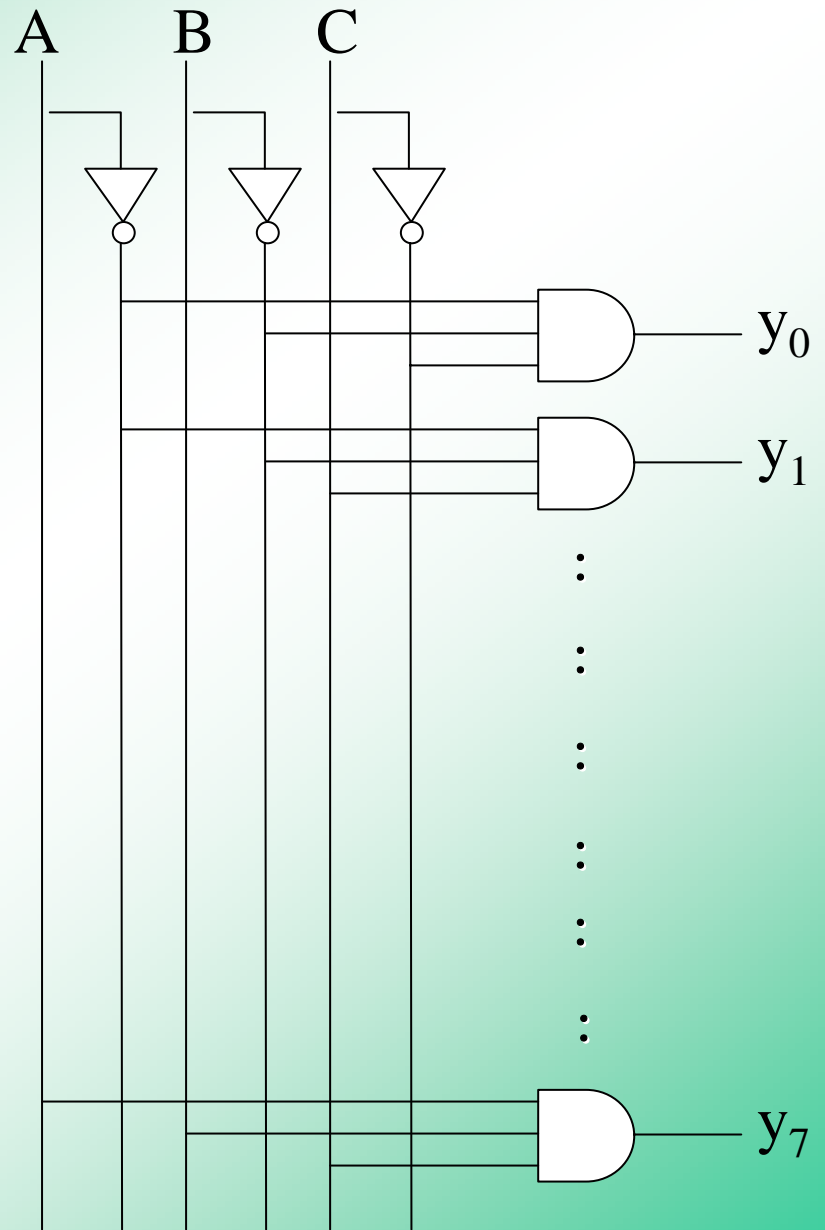
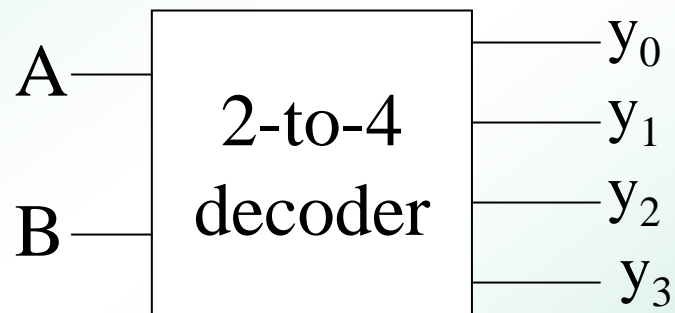
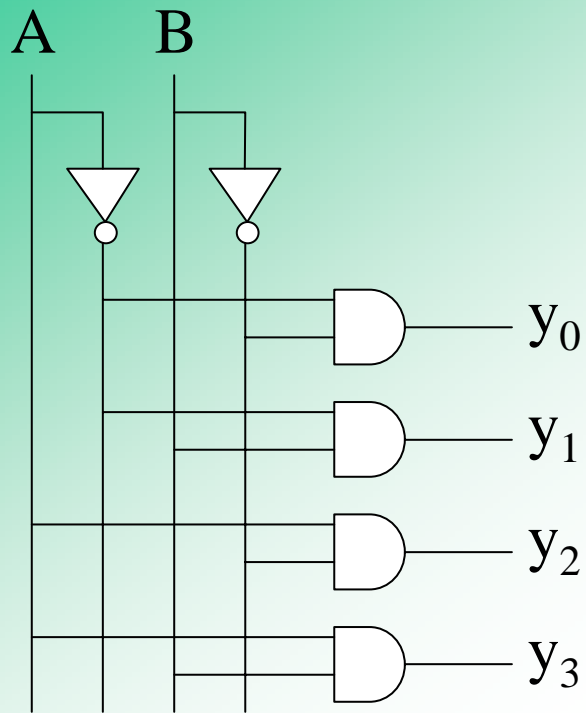
## 2. Chip I/O

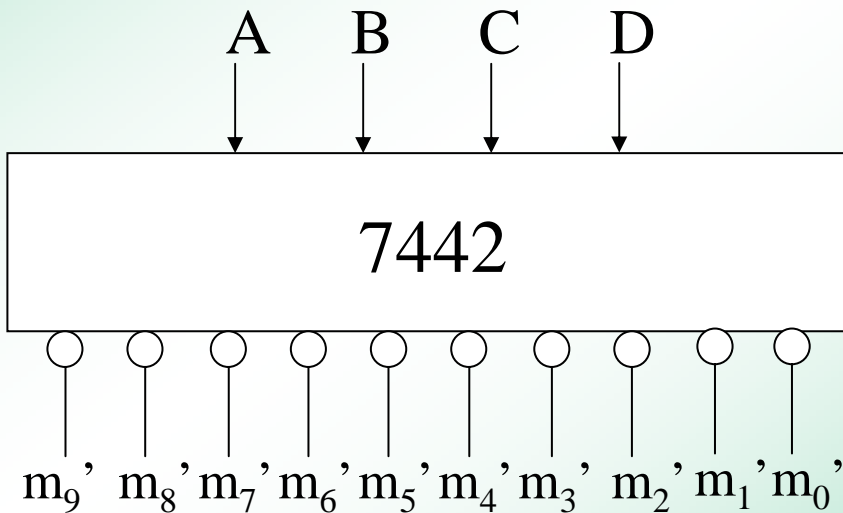
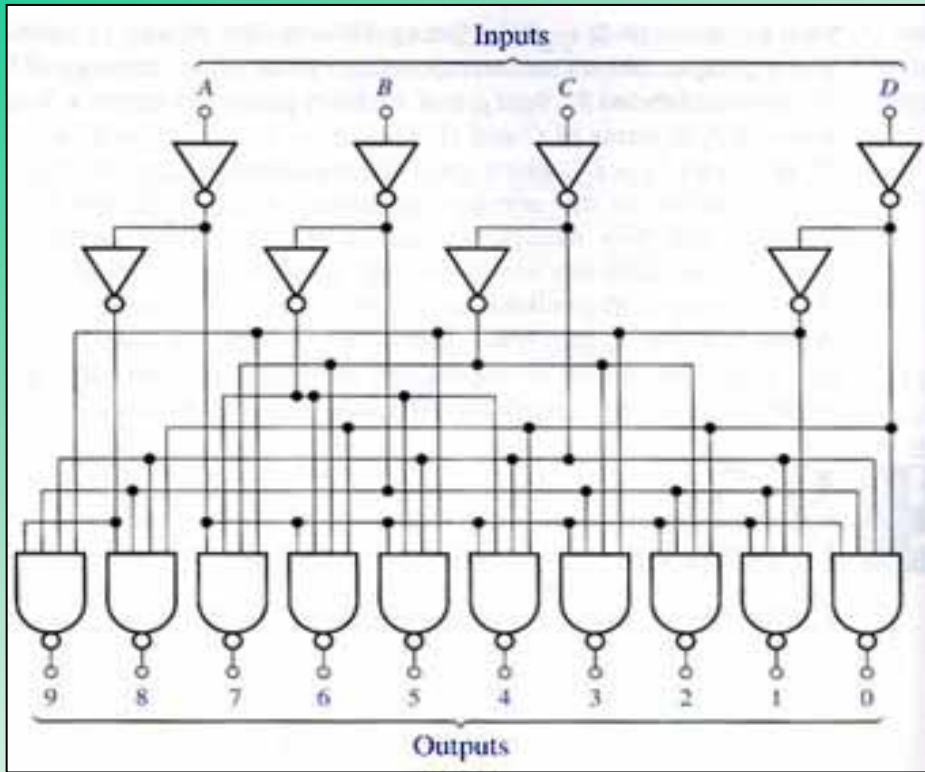


# § Decoders

Generate all of the minterms of inputs

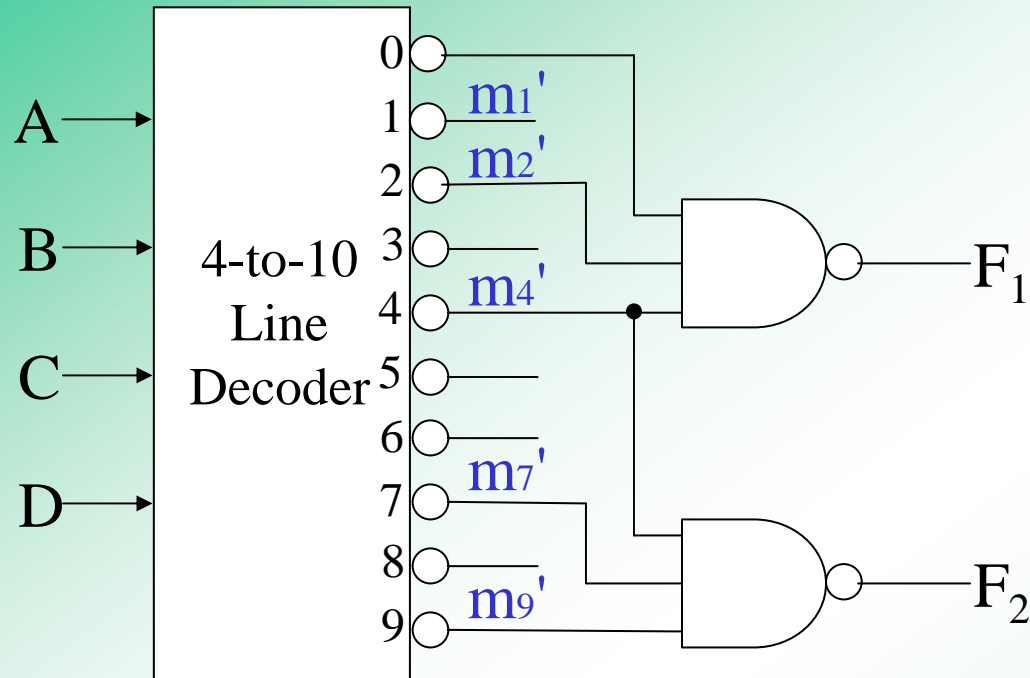






BCD input				Decimal Output									
A	B	C	D	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	1	1	0	1	1	1	1	1	1	1	1
0	0	1	0	1	1	0	1	1	1	1	1	1	1
0	0	1	1	1	1	1	0	1	1	1	1	1	1
0	1	0	0	1	1	1	1	0	1	1	1	1	1
0	1	0	1	1	1	1	1	1	0	1	1	1	1
0	1	1	0	1	1	1	1	1	1	0	1	1	1
0	1	1	1	1	1	1	1	1	1	1	0	1	1
1	0	0	0	1	1	1	1	1	1	1	1	0	1
1	0	0	1	1	1	1	1	1	1	1	1	1	0
1	0	1	0	1	1	1	1	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1

## Realization of a multiple output circuit using a decoder

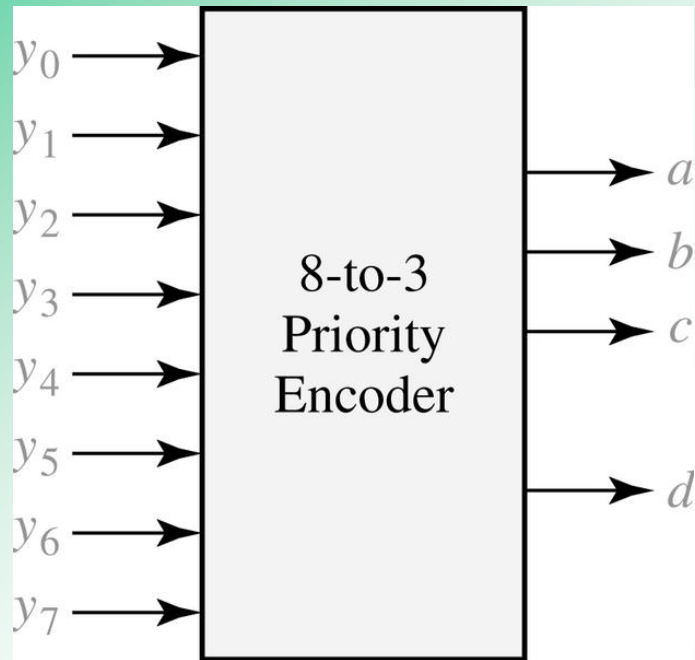


$$\text{又 } F_1 = m_1 + m_2 + m_4 \quad F_2 = m_4 + m_7 + m_9$$

可以 4 - to - 10 Decoder 來 realize

*By DeMorgan's law*

# § Encoders



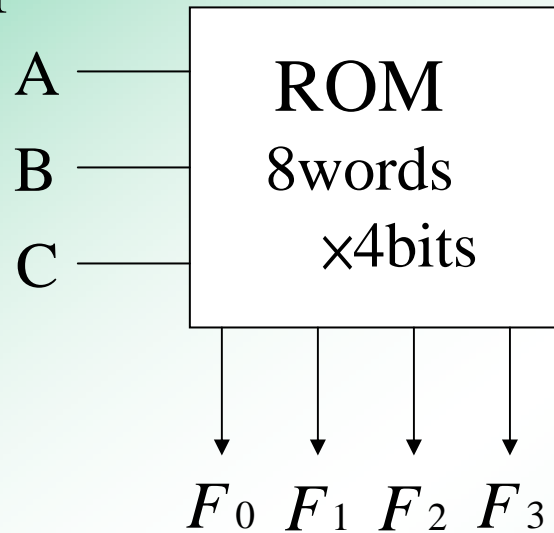
$y_0$	$y_1$	$y_2$	$y_3$	$y_4$	$y_5$	$y_6$	$y_7$	a	b	c	d
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	1
X	1	0	0	0	0	0	0	0	0	1	1
X	x	1	0	0	0	0	0	0	1	0	1
X	X	X	1	0	0	0	0	0	1	1	1
X	X	X	X	1	0	0	0	1	0	0	1
X	X	X	X	X	1	0	0	1	0	1	1
X	X	X	X	X	x	1	0	1	1	0	1
X	X	X	X	X	X	X	1	1	1	1	1

priority  
└─┬─┘

# § Read Only Memory

An LSI circuit to realize multiple output Boolean function(s)

Inputs



Outputs

A	B	C	$F_0$	$F_1$	$F_2$	$F_3$
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	0	1	0	0
0	1	1	0	1	1	0
1	0	0	0	1	1	0
1	0	1	1	1	1	0
1	1	0	0	0	1	1
1	1	1	1	1	1	1

Stored  
in ROM  
( $2^3$  words of  
4 bits each)

$$F_0 = m_0 + m_1 + m_5 + m_7$$

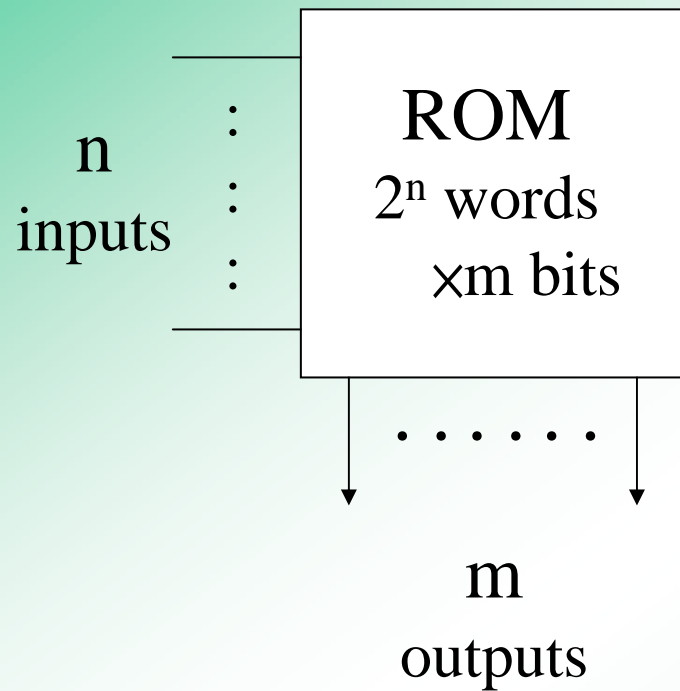
$$F_1 = m_2 + m_3 + m_4 + m_5 + m_7$$

$$F_2 = m_0 + m_1 + m_3 + m_4 + m_5 + m_6 + m_7$$

$$F_3 = m_6 + m_7$$

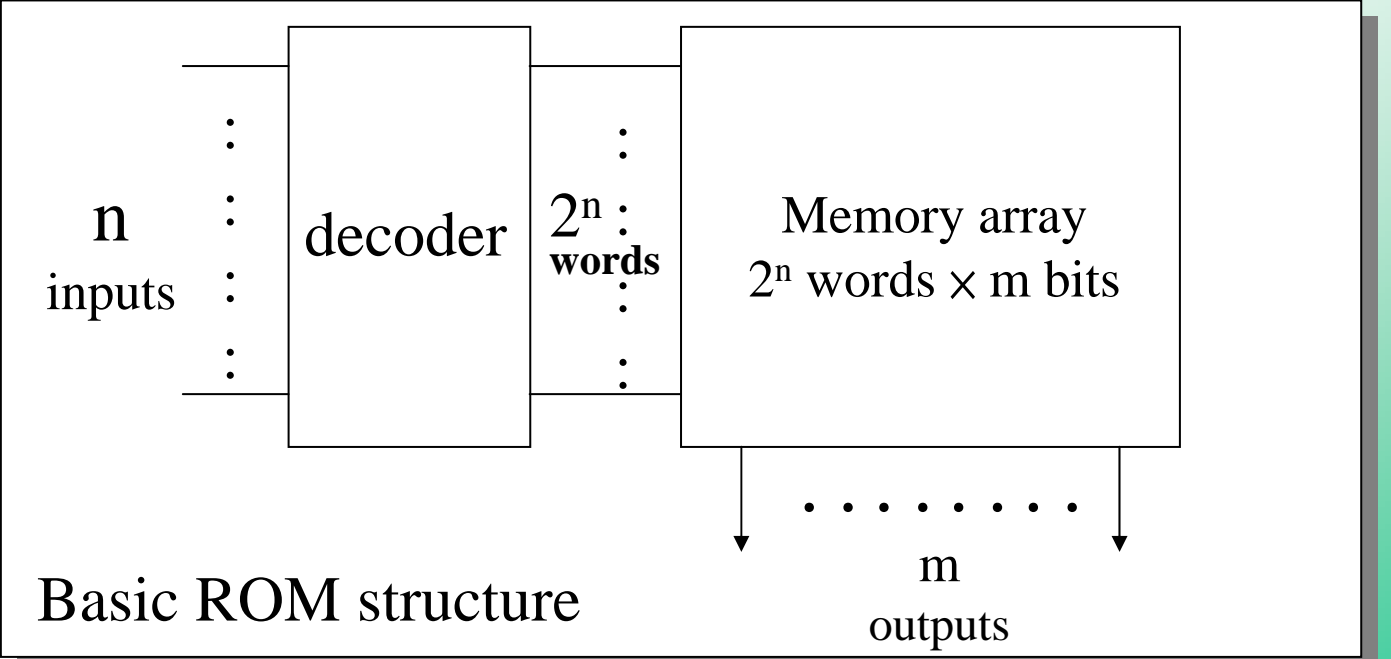
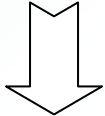
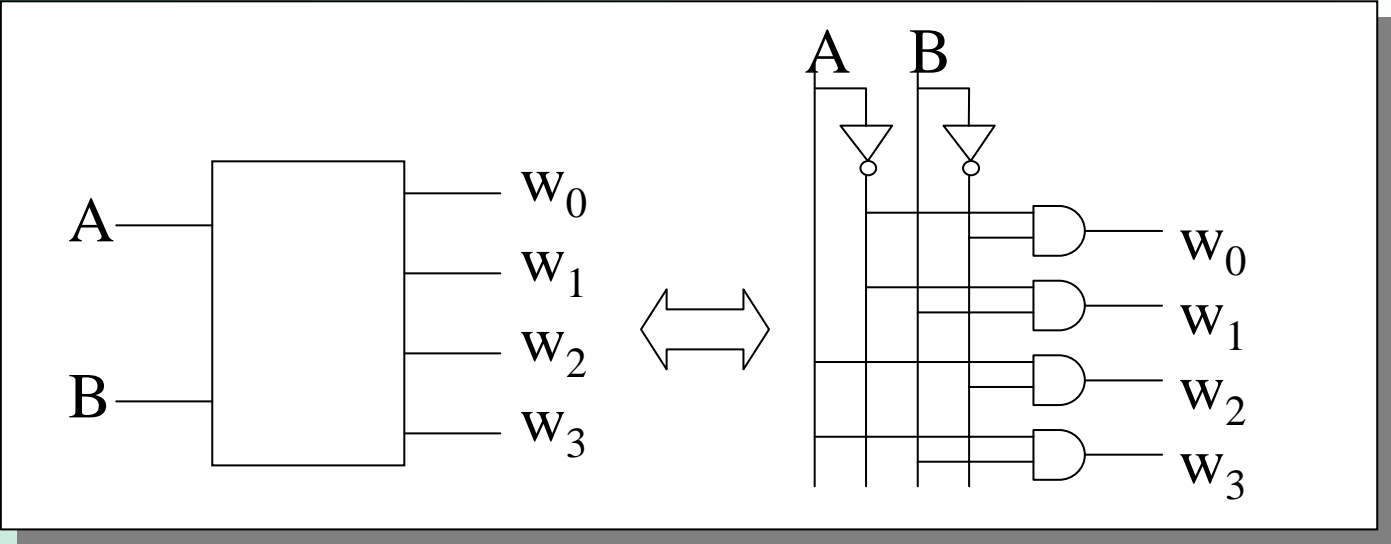


# General Form



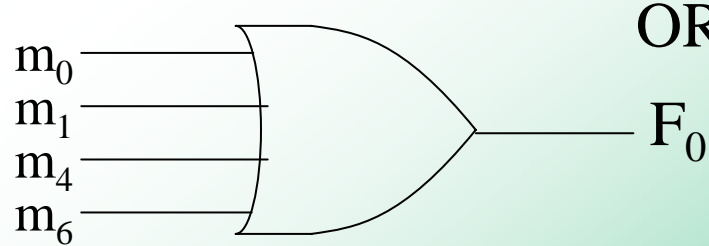
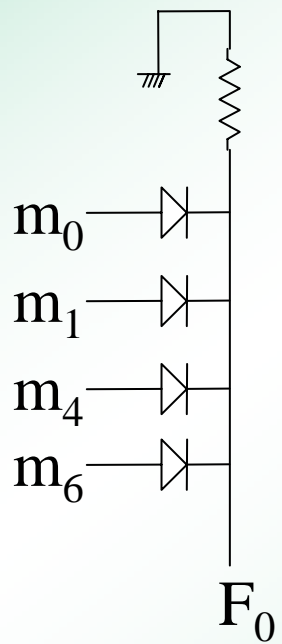
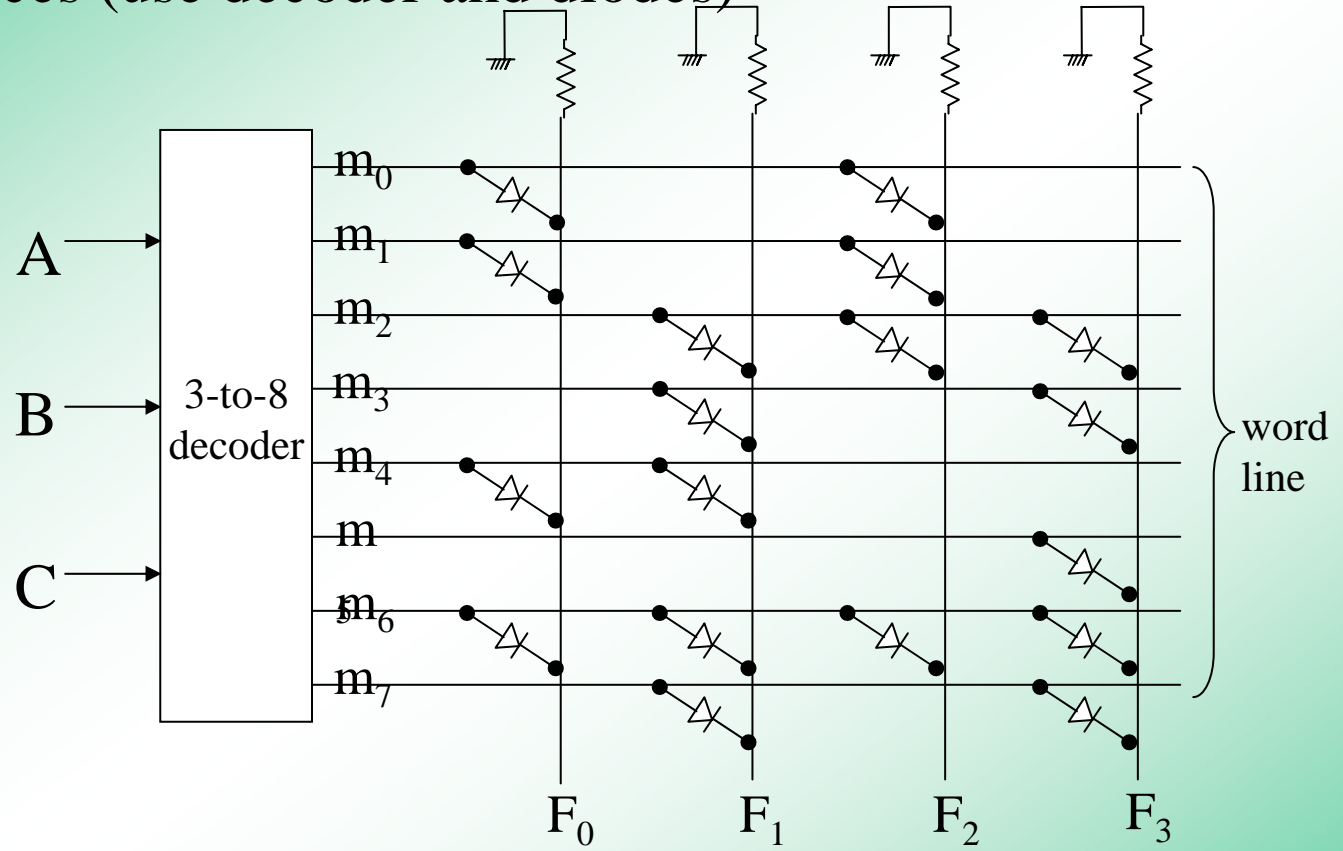
$n$ inputs				$m$ outputs				
				$F_0$	...	...	...	$F_{m-1}$
0	...	0	0	0	0	...	1	0
0	...	0	1	:				:
:			:	:				:
:			:	:				:
:			:	:				:
:			:	:				:
1	...	1	1	1	0	...	1	0

}  $2^n$  words



# ROM as logic devices (use decoder and diodes)

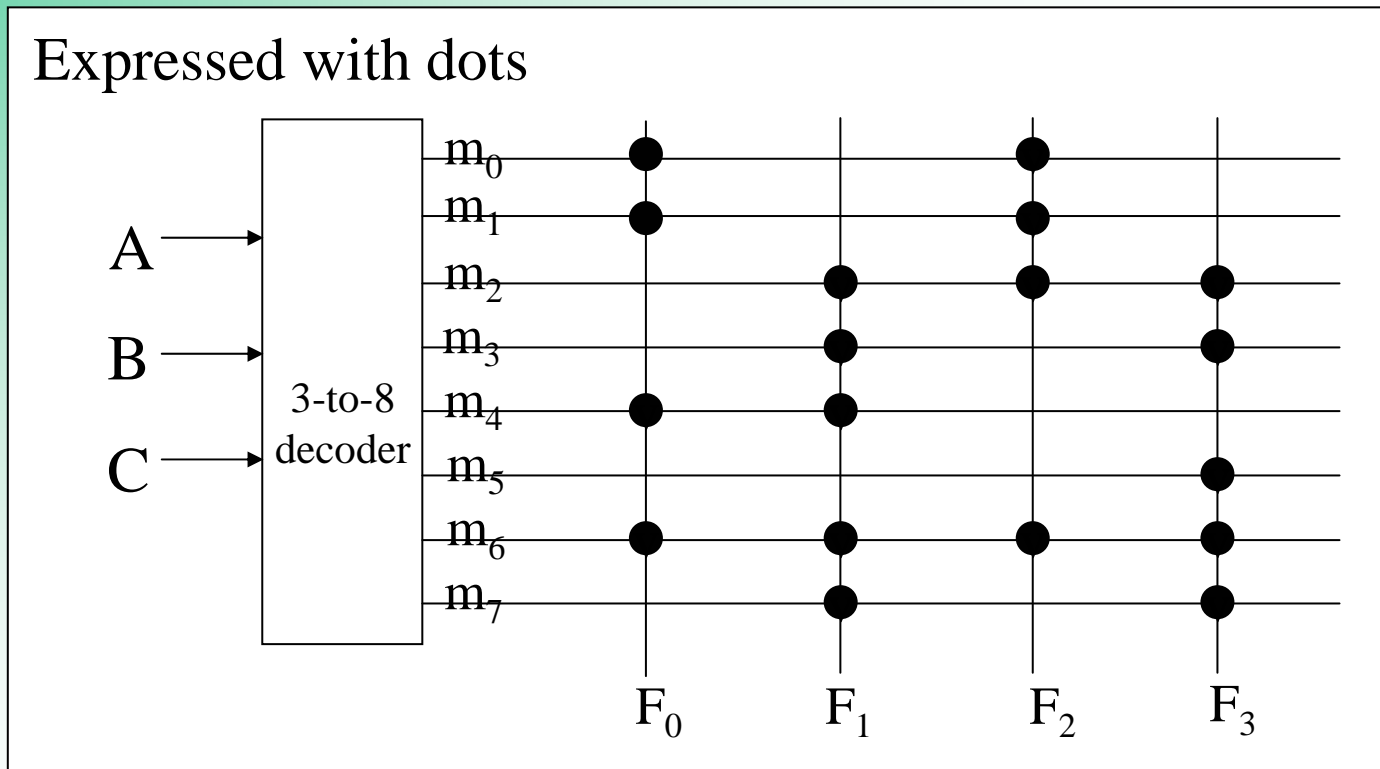
A	B	C	$F_0$	$F_1$	$F_2$	$F_3$
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	0	1	1	1
0	1	1	0	1	0	1
1	0	0	1	1	0	0
1	0	1	0	0	0	1
1	1	0	1	1	1	1
1	1	1	0	1	0	1



$$F_0 = m_0 + m_1 + m_4 + m_6$$

$$F_0 = \sum m(0,1,4,6) \quad F_1 = \sum m(2,3,4,6,7)$$

$$F_2 = \sum m(0,1,2,6) \quad F_3 = \sum m(2,3,5,6,7)$$



(1) Use "mask" to program ROM

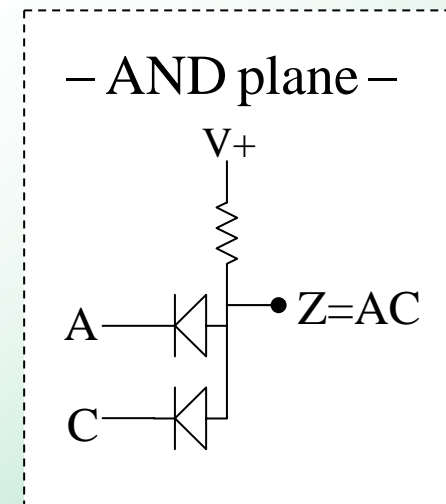
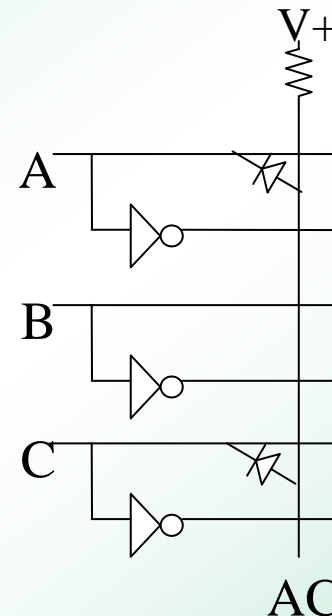
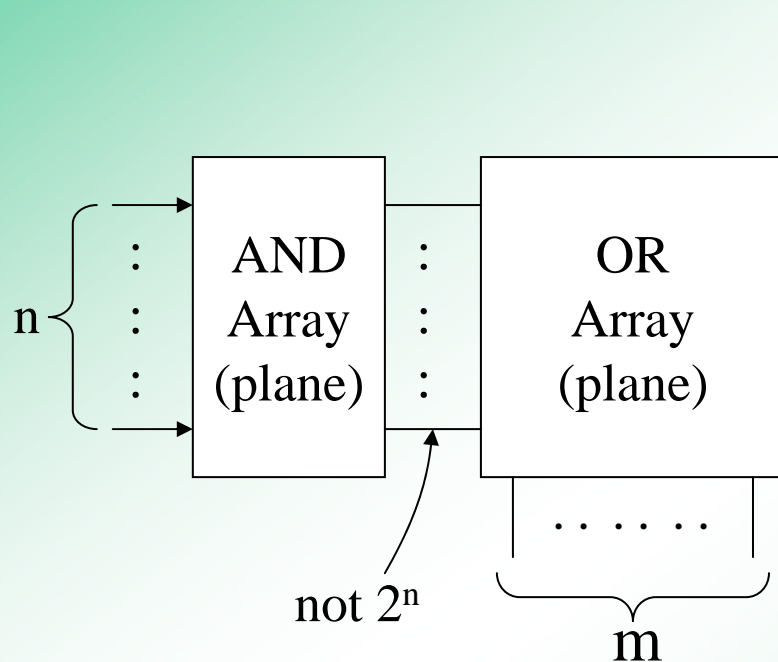
(2) EPROM electrically program UV light erase

(3) EEPROM

# § Programmable Logic Devices

Various kinds : PLA , PAL , EPLD , PEEL , GAL

※ PLAs :  $n \times m$  realizes  $m$  functions with  $n$  inputs.



2-level SOP implementation

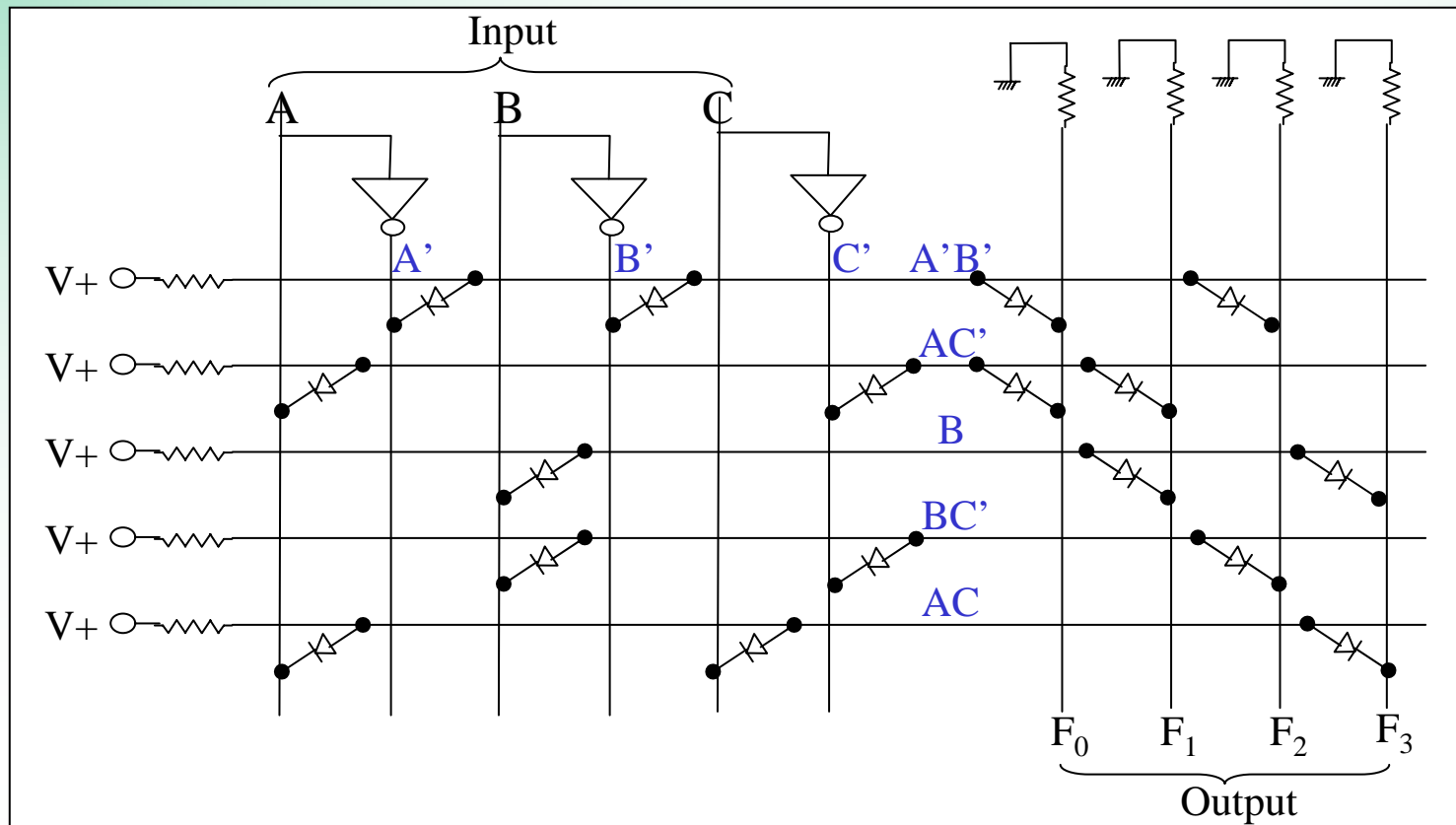
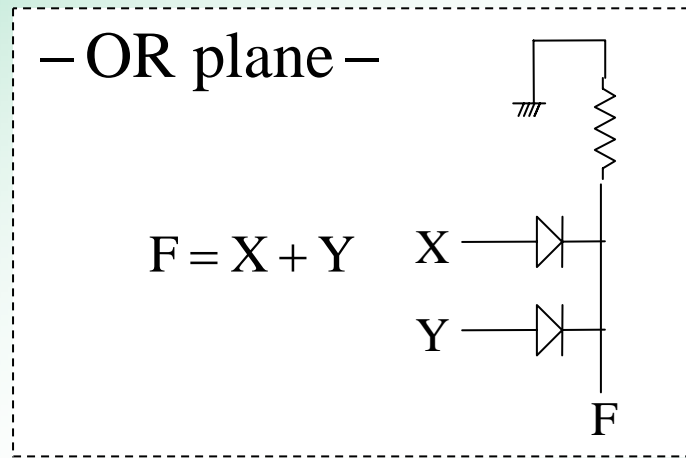
*ROM directly implement truth table*

If any of A , C not activated,

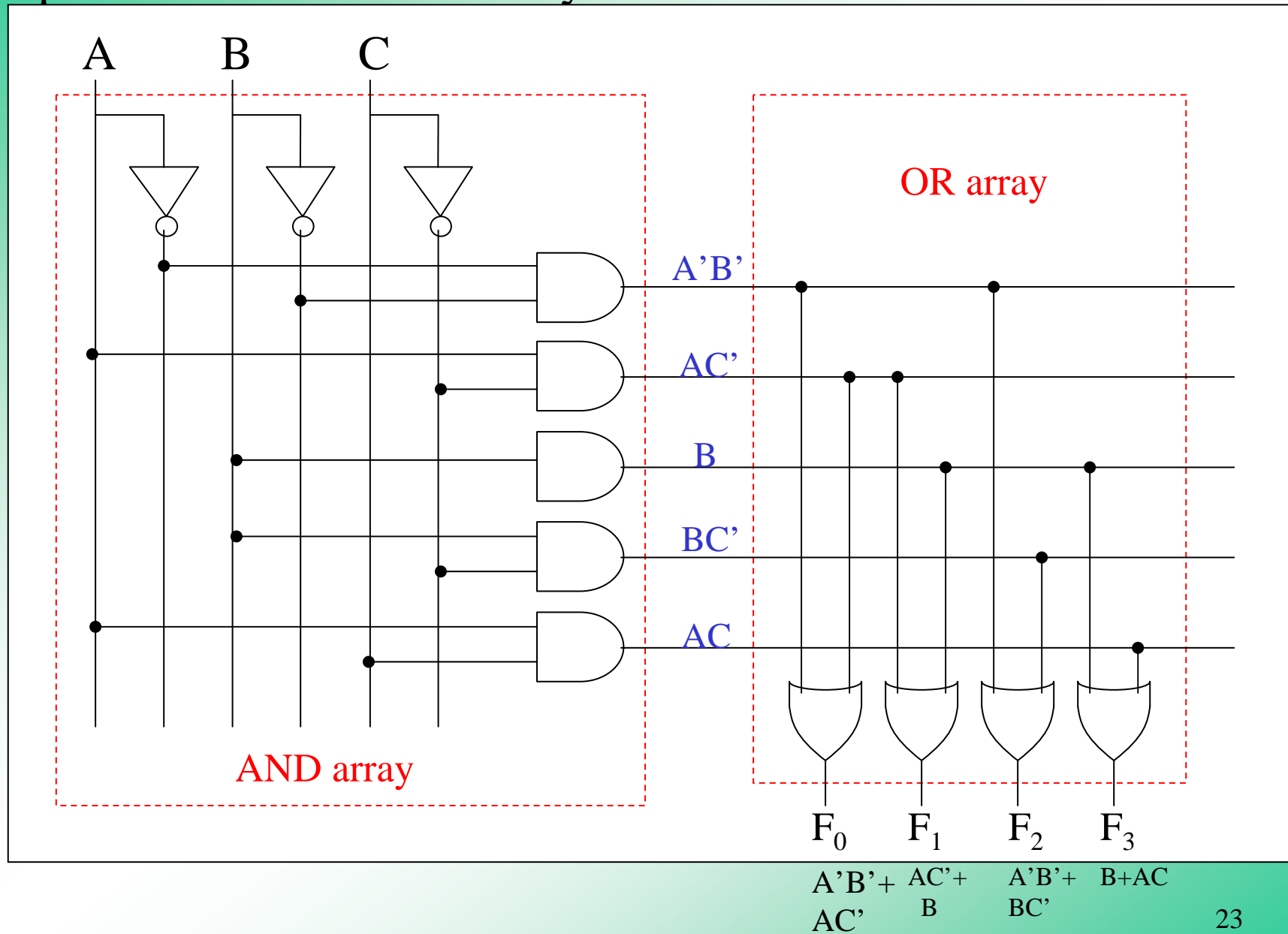
$A = C = \text{GND}$

i.e.  $A = 0$  or  $C = 0$

	A	B	C	F <sub>0</sub>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
A'B'	0	0	-	1	0	1	0
AC'	1	-	0	1	1	0	0
B	-	1	-	0	1	0	1
BC'	-	1	0	0	0	1	0
AC	1	-	1	0	0	0	1



# Equivalent AND - OR Array



Example

$$F_1 = \sum m(2,3,5,7,8,9,10,11,13,15)$$

$$F_2 = \sum m(2,3,5,6,7,10,11,14,15)$$

$$F_3 = \sum m(6,7,8,9,13,14,15)$$

Minimized multiple output expressions (using K - map)

$$F_1 = a'bd + abd + ab'c' + b'c$$

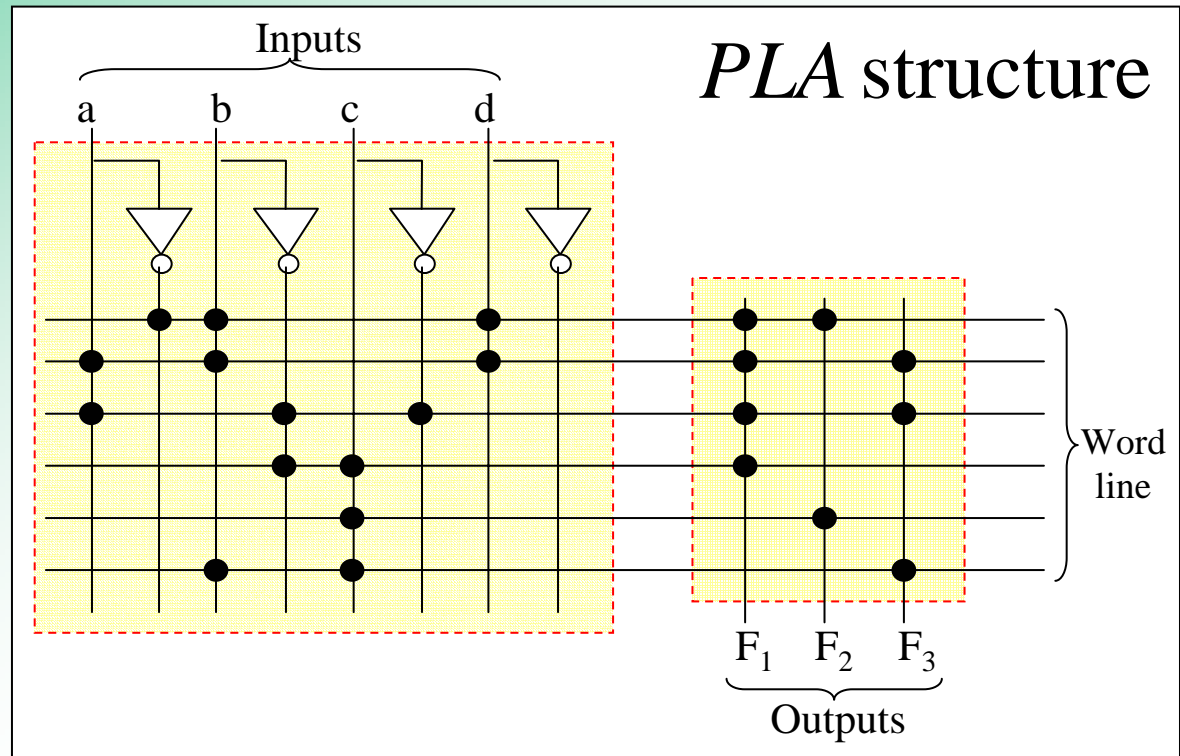
$$F_2 = c + a'bd$$

$$F_3 = bc + ab'c' + abd$$

	a	b	c	d	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
<i>PLA table</i>	0	1	–	1	1	1	0
	1	1	–	1	1	0	1
	1	0	0	–	1	0	1
	–	0	1	–	1	0	0
	–	–	1	–	0	1	0
	–	1	1	–	0	0	1



a	b	c	d	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
0	1	-	1	1	1	0
1	1	-	1	1	0	1
1	0	0	-	1	0	1
-	0	1	-	1	0	0
-	-	1	-	0	1	0
-	1	1	-	0	0	1



a b c d = 0 1 1 1    1,5,6 rows selected,  $\Rightarrow F_1 = 1(1\text{st row}) + 0(5\text{th}) + 0(6\text{th}) = 1$

$$F_2 = 1 + 1 + 0 = 1$$

$$F_3 = 0 + 0 + 1 = 1$$

= 0 0 0 1    no rows are selected,  $\Rightarrow F_1F_2F_3 = 000$

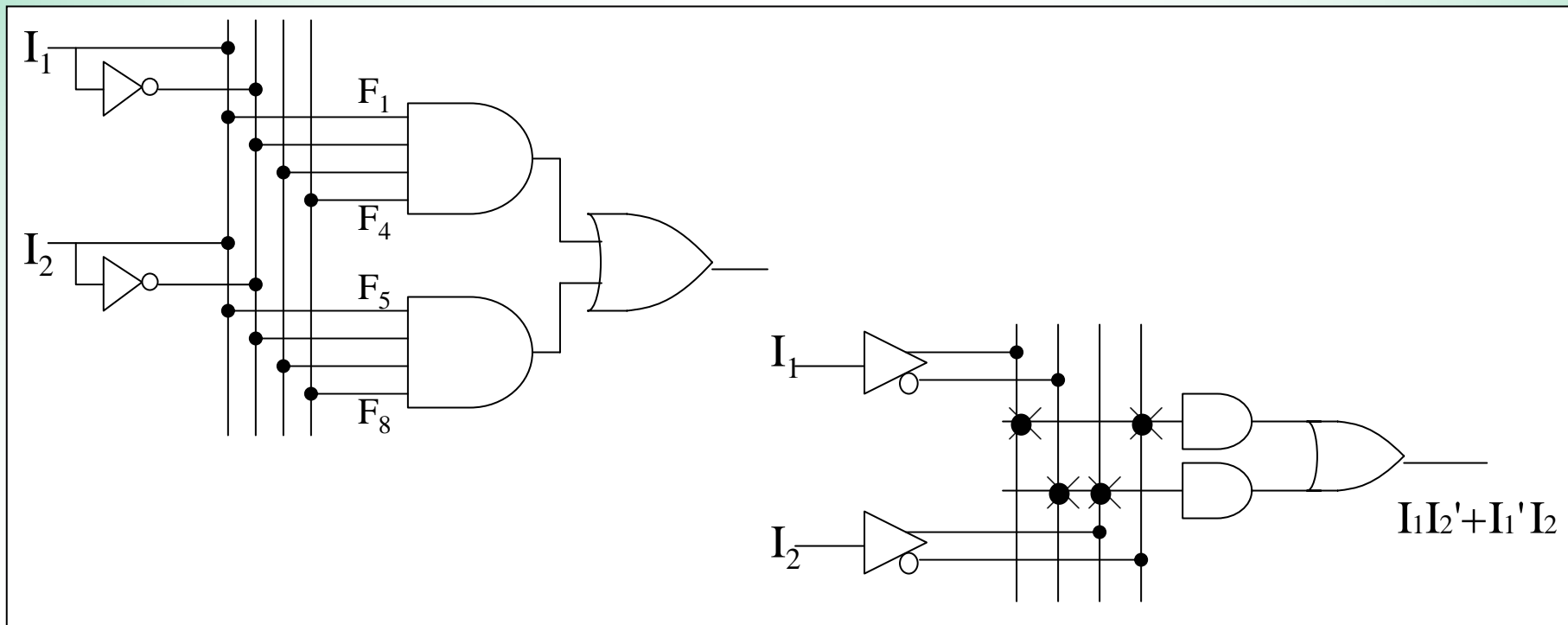
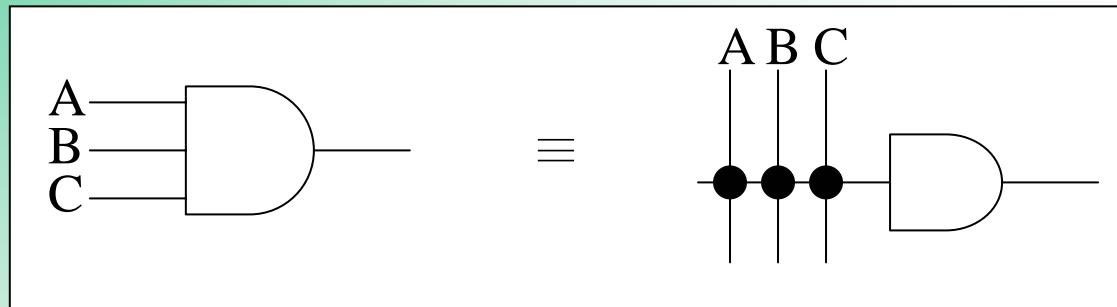
= 1 0 0 1    3rd rows selected,  $\Rightarrow F_1F_2F_3 = 101$

© Mask programmable PLA

© Field programmable PLA

❖ PALs :  $n \times m$  realizes  $m$  functions with  $n$  inputs.

A special case of PLA.  $\left( \begin{array}{l} \text{AND array programmable} \\ \text{OR array NOT programmable} \end{array} \right)$

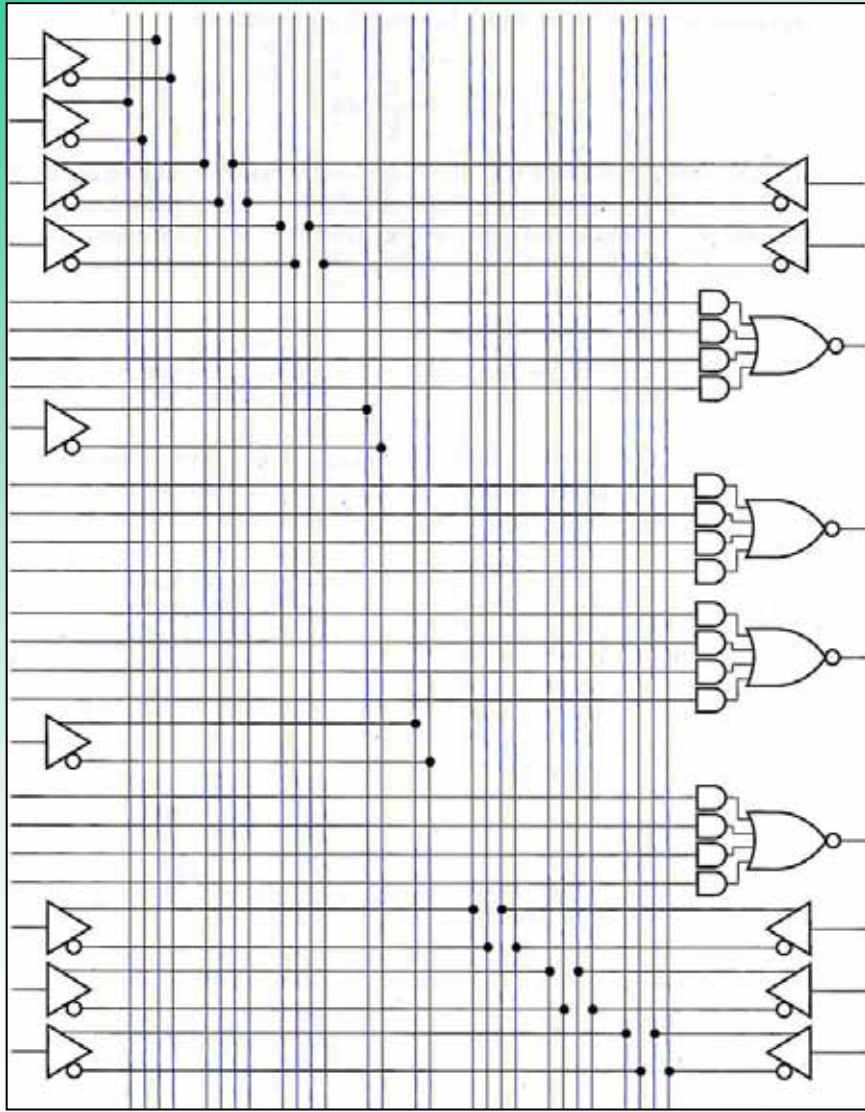


## Some typical available PALs

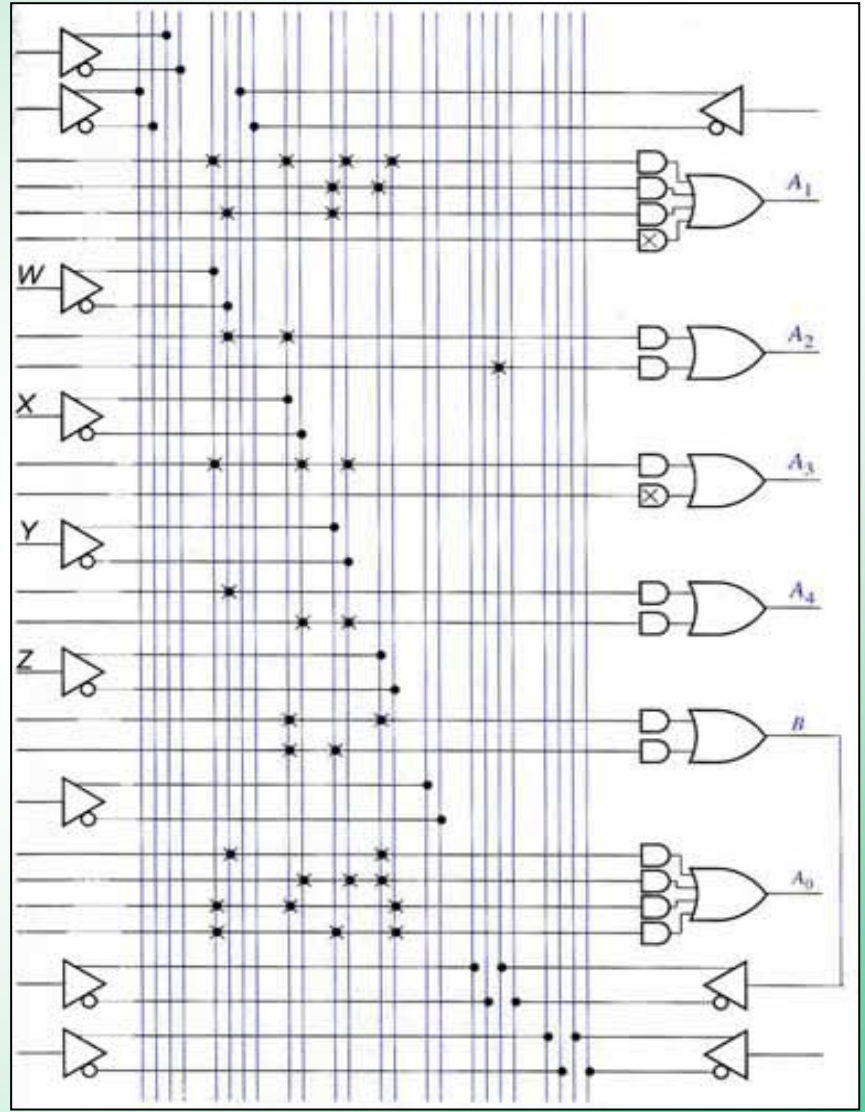
Type No.*	No. of Inputs	No. of Outputs	Gate Configuration	No. of Inputs per OR GATE
10H8	10	8	AND - OR	2
12H6	12	6	AND - OR	4,2,2,2,2,4
14H4	14	4	AND - OR	4
16H2	16	2	AND - OR	8
16C1	16	1	AND - OR/NOR	16
20C1	20	1	AND - OR/NOR	16
10L8	10	8	AND - NOR	2
12L6	12	6	AND - NOR	4,2,2,2,2,4
14L4	14	4	AND - NOR	4
16L2	16	2	AND - NOR	8
12L10	12	10	AND - NOR	2
14L8	14	8	AND - NOR	4,2,2,2,2,2,4
16L6	16	6	AND - NOR	4,4,2,2,2,4
18L4	18	4	AND - NOR	4
20L2	20	2	AND - NOR	8
16L8	16	8	AND - NOR	8
20L8	20	8	AND - NOR	8
20L10	20	10	AND - NOR	4

H	active H
L	active L
C	both active H and L

# Internal Logic diagram



**14L4**



**12H6**

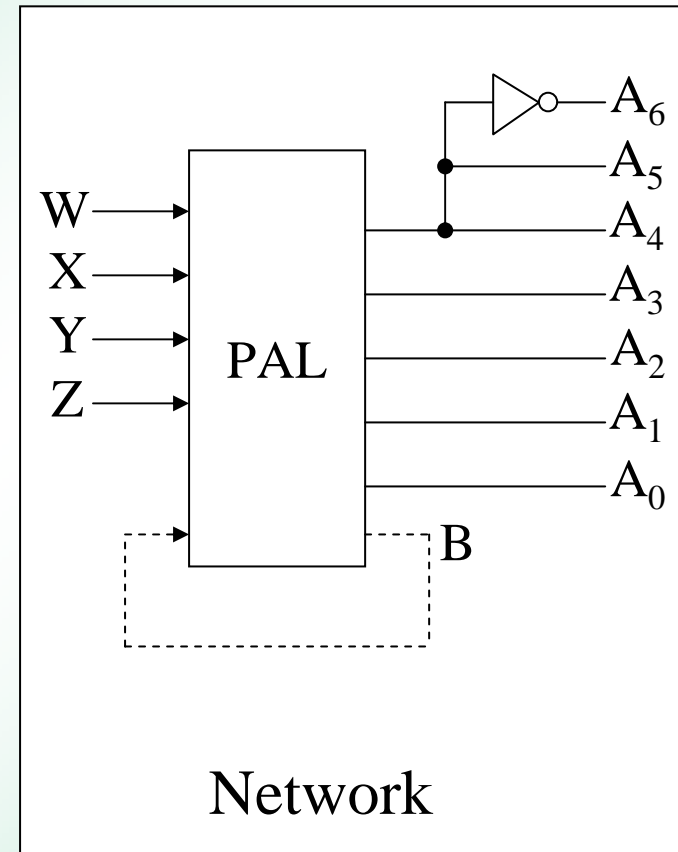
Number of inputs to an OR gate fixed and limited.

# Example

## Hexadecimal Digit : Binary to ASC II Converter

Input				Hex	ASC II Code for Hex Digit						
W	X	Y	Z	Digit	A <sub>6</sub>	A <sub>5</sub>	A <sub>4</sub>	A <sub>3</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>
0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	1	1	0	1	1	0	0	0	1
0	0	1	0	2	0	1	1	0	0	1	0
0	0	1	1	3	0	1	1	0	0	1	1
0	1	0	0	4	0	1	1	0	1	0	0
0	1	0	1	5	0	1	1	0	1	0	1
0	1	1	0	6	0	1	1	0	1	1	0
0	1	1	1	7	0	1	1	0	1	1	1
1	0	0	0	8	0	1	1	1	0	0	0
1	0	0	1	9	0	1	1	1	0	0	1
1	0	1	0	A	1	0	0	0	0	0	1
1	0	1	1	B	1	0	0	0	0	1	0
1	1	0	0	C	1	0	0	0	0	1	1
1	1	0	1	D	1	0	0	0	1	0	0
1	1	1	0	E	1	0	0	0	1	0	1
1	0	1	1	F	1	0	0	0	1	1	0

Truth table



$$A_4 = A_5 \quad A_6 = \overline{A_5}$$

∴ 4 input variables } ⇒ 12H6 or 12L6  
 5 output functions }

Use K - map method :

$$\Rightarrow A_4 = W' + X'Y' \quad A_3 = WX'Y' \quad A_2 = W'X + XZ + XY$$

$$A_4' = WX + WY \quad A_3' = \underline{\underline{W'}} + \underline{\underline{X}} + \underline{\underline{Y}} \quad A_2' = X' + WY'Z'$$

$$A_1 = WXY'Z' + YZ + W'Y \quad A_0 = \underline{\underline{W'Z}} + \underline{\underline{X'Y'Z}} + \underline{\underline{WXZ'}} + \underline{\underline{WYZ'}}$$

$$A_1' = W'Y' + Y'Z + X'Y' + WYZ' \quad A_0' = W'Z' + WYZ + WX'Z + X'Y'Z$$

$$A_0: \left. \begin{array}{l} 1 \times 4 \text{ inputs AND to OR} \\ 2 \times 3 \text{ inputs AND to OR} \end{array} \right\} 12H6 \text{ not available}$$

但  $A_2 = W'X + (XZ + XY) = W'X + B$

B connected back to one input

∴ 12H6 結果可用

CAD programs are available.

# PLD Summary

AND Array

OR Array

PLA

Programmable

Programmable

PAL

Programmable

Fixed

ROM

Fixed

Programmable

Not  
Programmable

Fixed

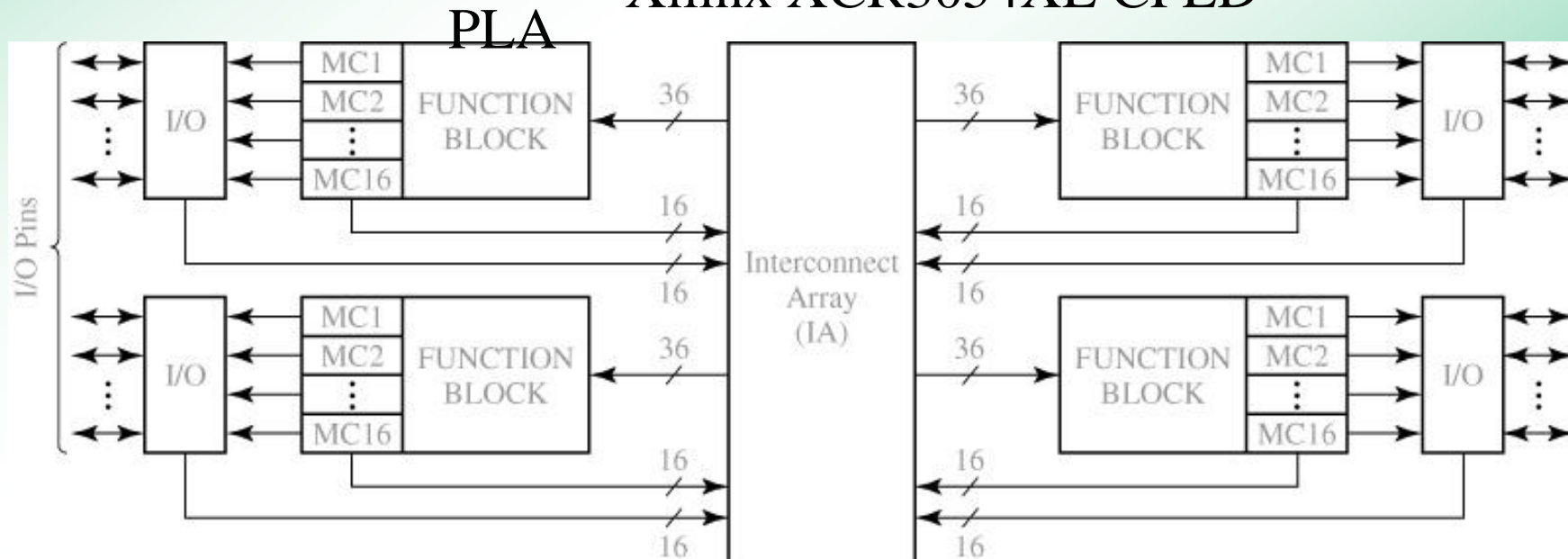
Fixed

# § Complex Programmable Logic Devices (CPLD)

Integrate and interconnect many PALs and PLAs on a single chip

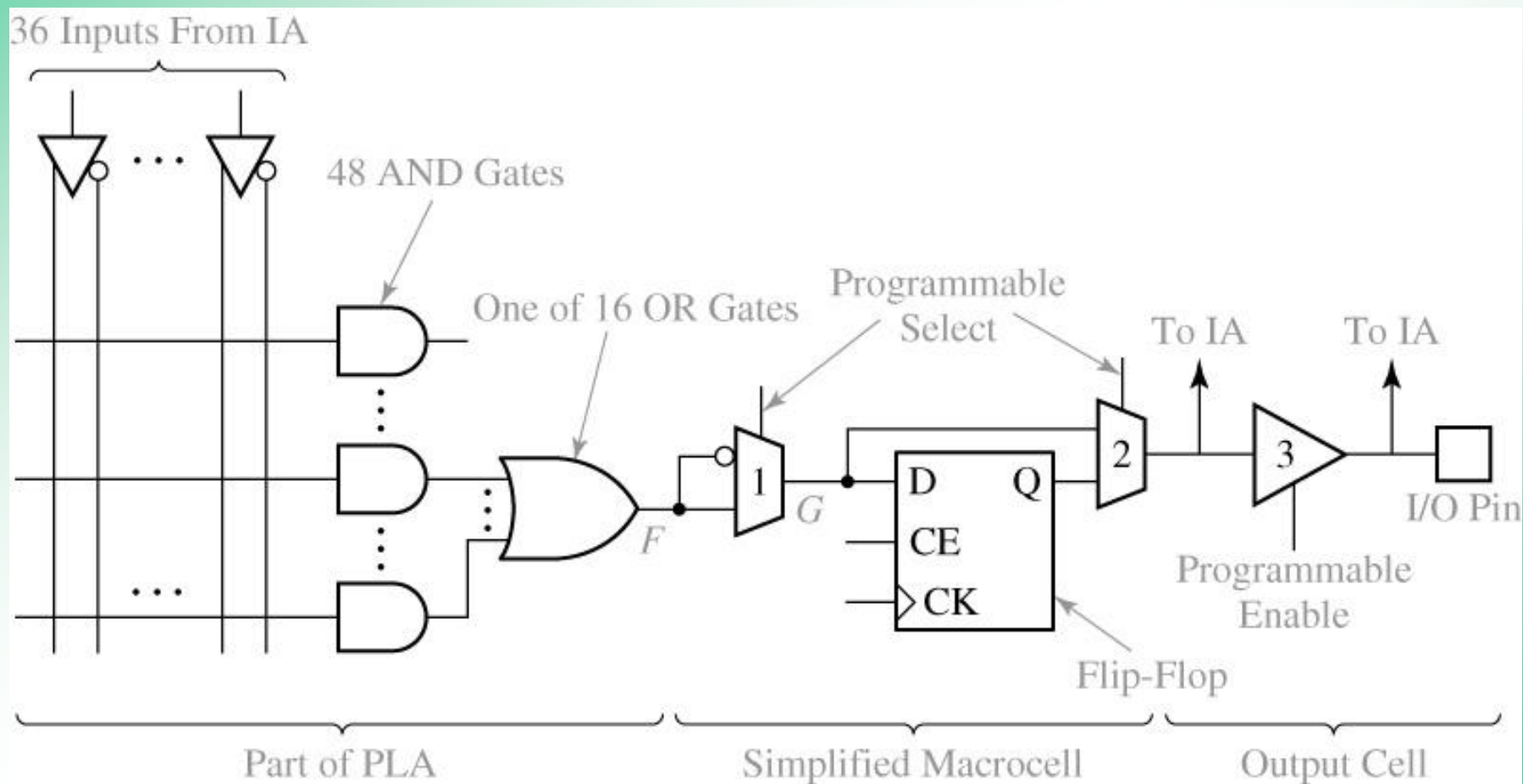
- *PLA to form combinational logic*
- *I/O Cell*
- *Macro Cell*
  - *Register for Sequential Logic*
  - *Pass Through for Combinational Logic*
  - *Three State Buffer for Bidirectional I/O*
  - *Input Latch to synchronize inputs*
- *Floating Gate Programmable*

Xilinx XCR3054XL CPLD





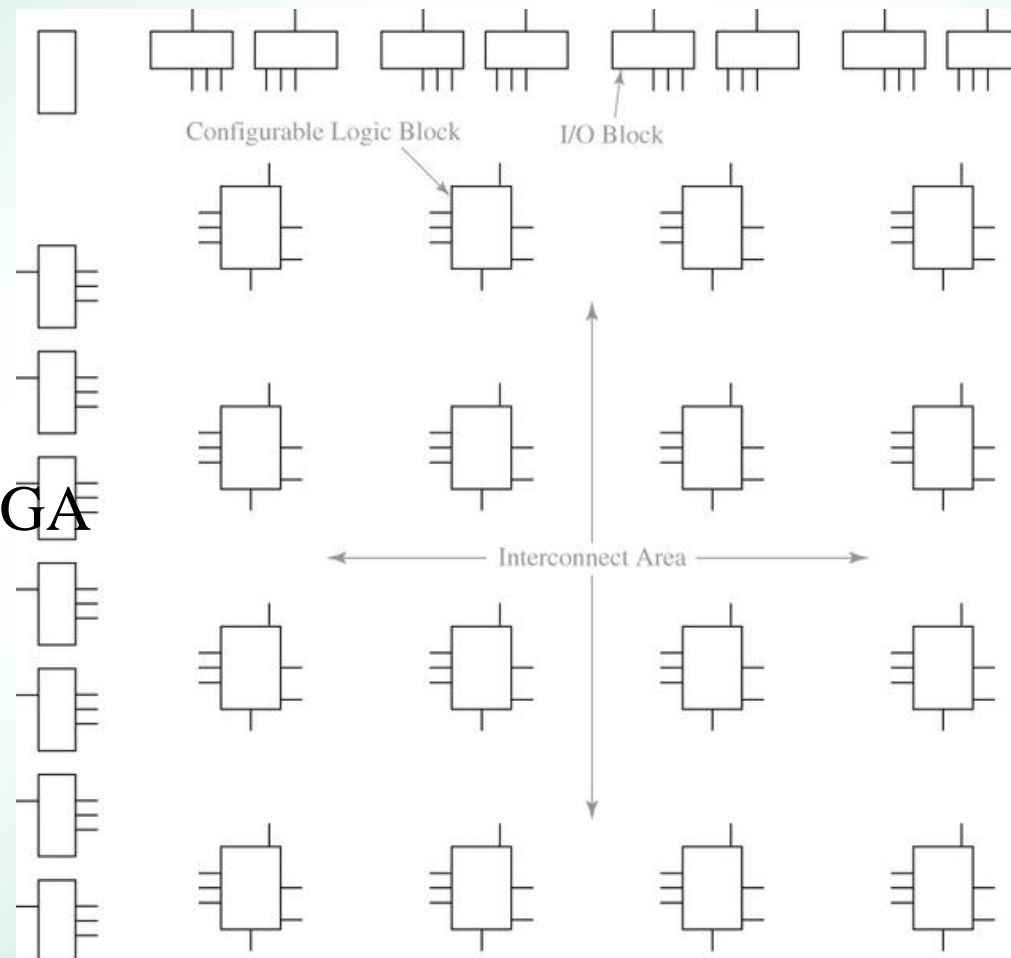
## CPLD Function block an macrocell



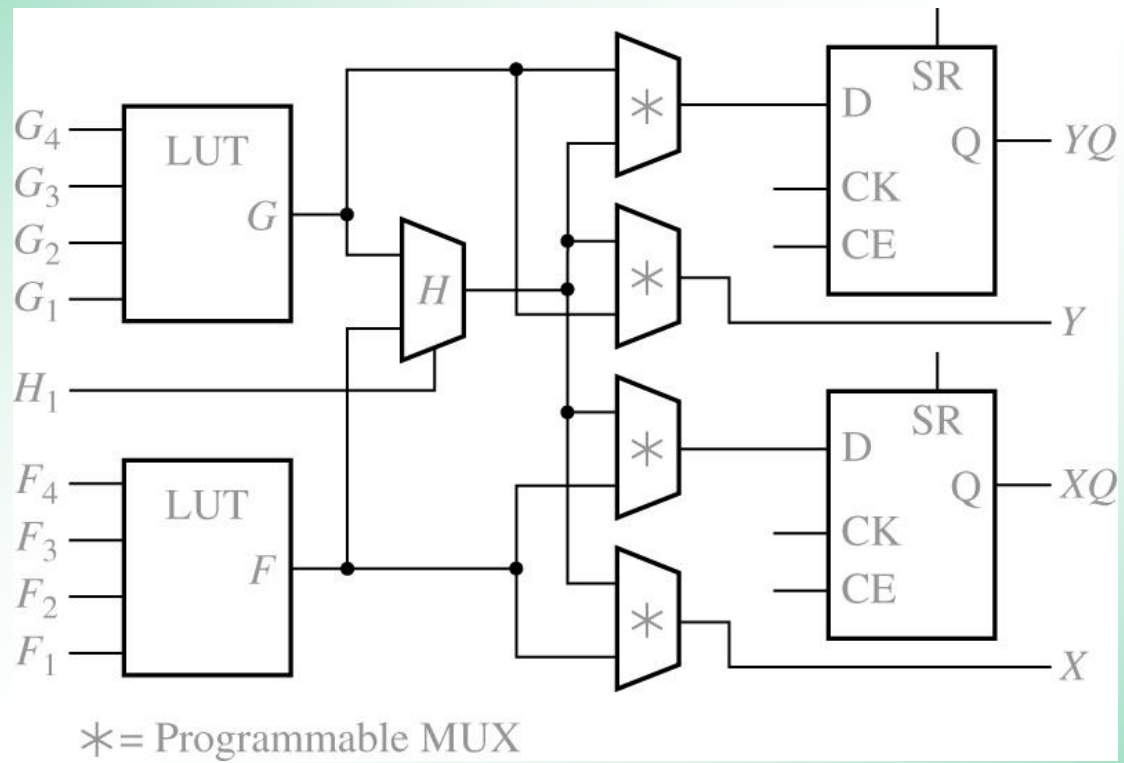
# § Field Programmable Gate Array (FPGA)

An IC that contains an array of identical logic cells with programmable interconnections

Layout of a typical FPGA



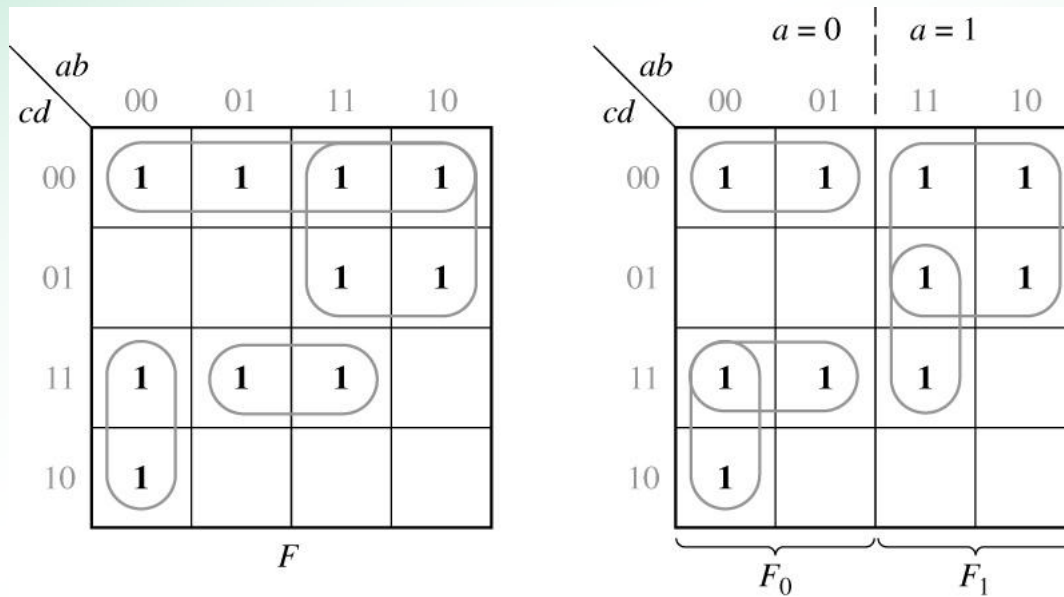
## CLB: configurable logic block



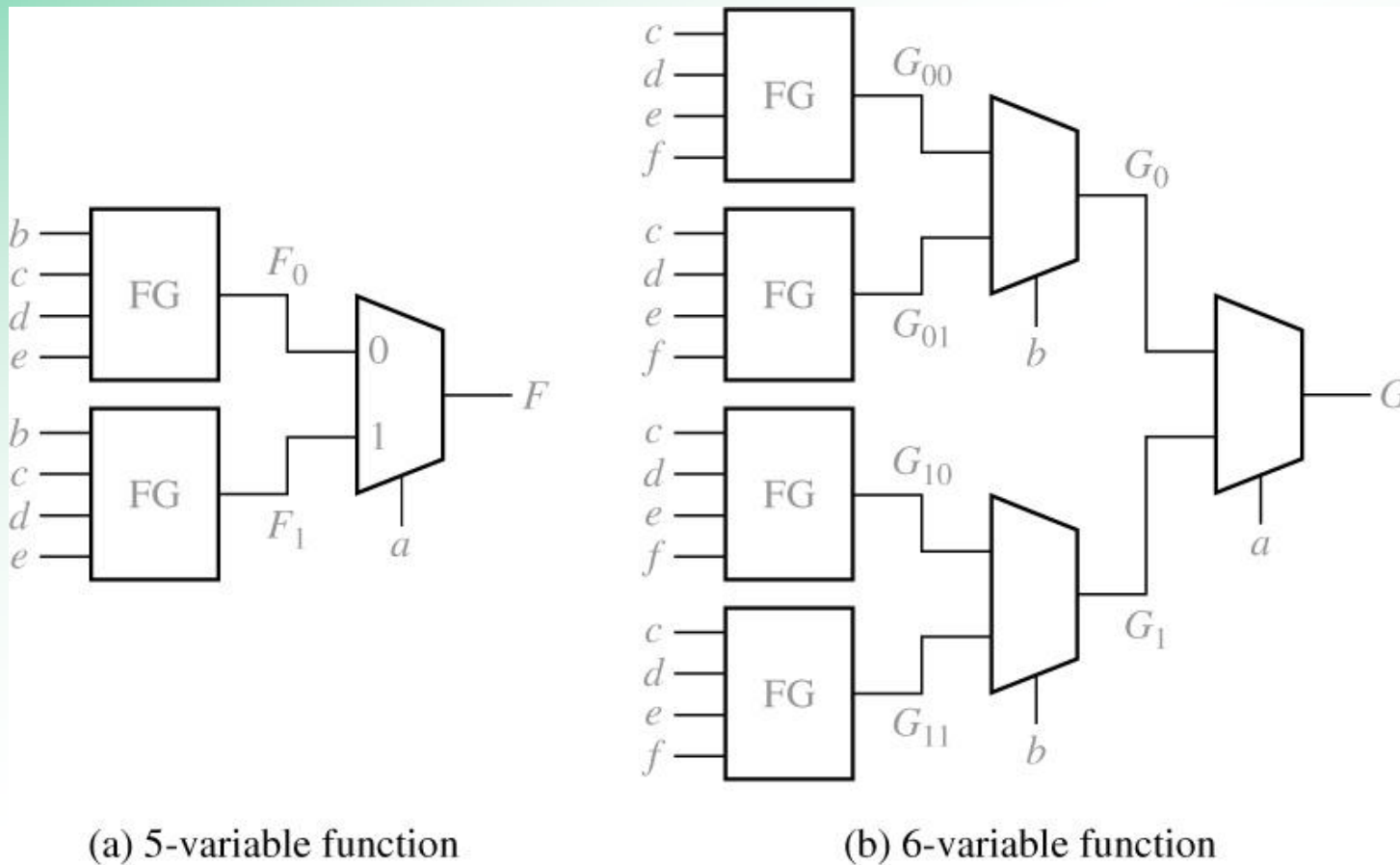
# Decomposition of switching functions

## Shannon's expansion theorem

$$F(x_1, x_2, \dots, x_{i-1}, x_i, x_{i+1}, \dots) \\ = x_i' f_0 + x_i f_1$$



## Realization of 5- and 6-variables functions with function generators



# Homework of Chap.9

12.(a)

15.

21.

23.

26.

30.