Unit 6 Quine – McClusky Method

Systematic way to find nearly minimum SOP form of a function from minterm expression.

Two steps：

1. Determine Prime Implicants
2. Find minimum set of prime implicants to express the function

Step 1 *Determination of Prime Expression : to obtain minimal number of literals*

利用 $XY + XY' = X$

$AB'CD' + AB'CD = AB'C$

$1\ 0\ 1\ 0 \quad 1\ 0\ 1\ 1 = 1\ 0\ 1-$

$A'BC'D + A'BCD'$ won't

$0\ 1\ 0\ 1 \quad 0\ 1\ 1\ 0$ combine

Example $F(a,b,c,d) = \sum m(0,1,2,5,6,7,8,9,10,14)$

(1)

Group 0	0	0	0	0	0
Group 1	1	0	0	0	1
	2	0	0	1	0
	8	1	0	0	0
Group 2	5	0	1	0	1
	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
Group 3	7	0	1	1	1
	14	1	1	1	0

依照1之個數分成groups

cd \ ab				
	1			1
	1	1		1
		1		
	1	1	1	1

0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

(2) Build Table 6－1

Group 0	0	0000	∨
	1	0001	∨
Group 1	2	0010	∨
	8	1000	∨
	5	0101	∨
Group 2	6	0110	∨
	9	1001	∨
	10	1010	∨
Group 3	7	0111	∨
	14	1110	∨

0,1	000 －	∨
0,2	00 － 0	∨
0,8	－ 000	∨
1,5	0 － 01	
1,9	－ 001	∨
2,6	0 － 10	∨
2,10	－ 010	∨
8,9	100 －	∨
8,10	10 － 0	∨
5,7	01 － 1	
6,7	011 －	
6,14	－ 110	∨
10 ,14	1 － 10	∨

0,1,8,9	－ 00 －
0,2,8,10	－ 0 － 0
0,8,1,9	－ 00 －
0,8,2,10	－ 0 － 0
2,6,10 ,14	－ － 10
2,10 ,6,14	－ － 10

比較 Groups 0 , 1
Groups 1 , 2
Groups 2 , 3
. . . .
打鉤∨

比較
同有一其他
只相差 1 之
terms
可重覆用
打鉤∨

Cross - out
相同
redundant terms
再進行如左
之procedure

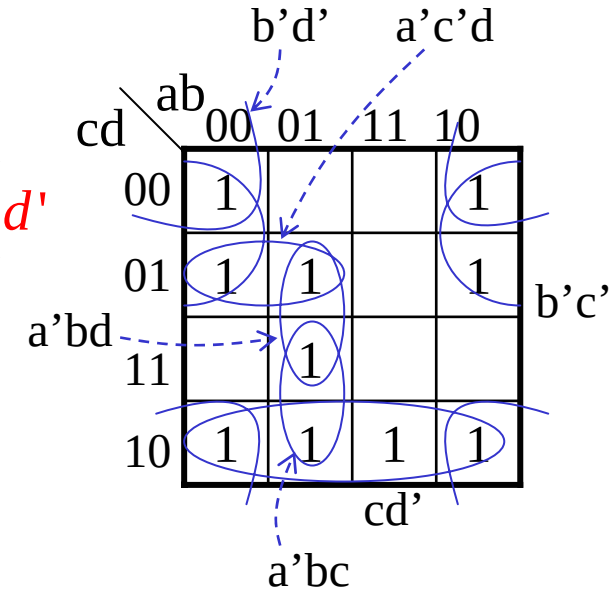
最終：沒有打鉤之implicant即爲prime implicants

$F = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$

(1,5) (5,7) (6,7) (0,1,8,9) (0,2,8,10) (2,6,10,14)

$F = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$

$= a'bd + b'c' + cd'$



Step 2 Prime Implicant Chart : to find minimum number of prime implicants to express F

		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	b'c'	×	×					×	⊗		
(0,2,8,10)	b'd'	×		×				×		×	
(2,6,10,14)	cd'			×		×				×	⊗
(1,5)	a'c'd		×		×						
(5,7)	a'bd				×		×				
(6,7)	a'bc					×	×				

(Find columns containing one × ⇒ b'c' and cd' are "essential")

		0	1	2	5	6	7	8	9	10	14
(0,1,8,9)	b'c'	×	×					×	×		
(0,2,8,10)	b'd'	×		×				×		×	
(2,6,10,14)	cd'			×		×				×	×
(1,5)	a'c'd		×		×						
(5,7)	a'bd				×		×				
(6,7)	a'bc					×	×				

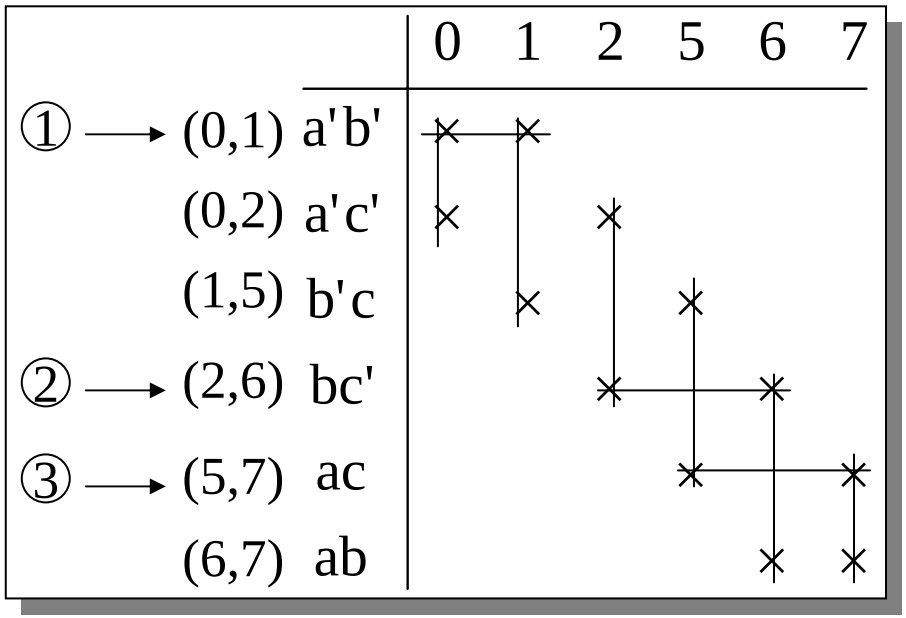
("covers" $F = b'c' + cd' + a'bd$)

有時，can't find columns containing one ×

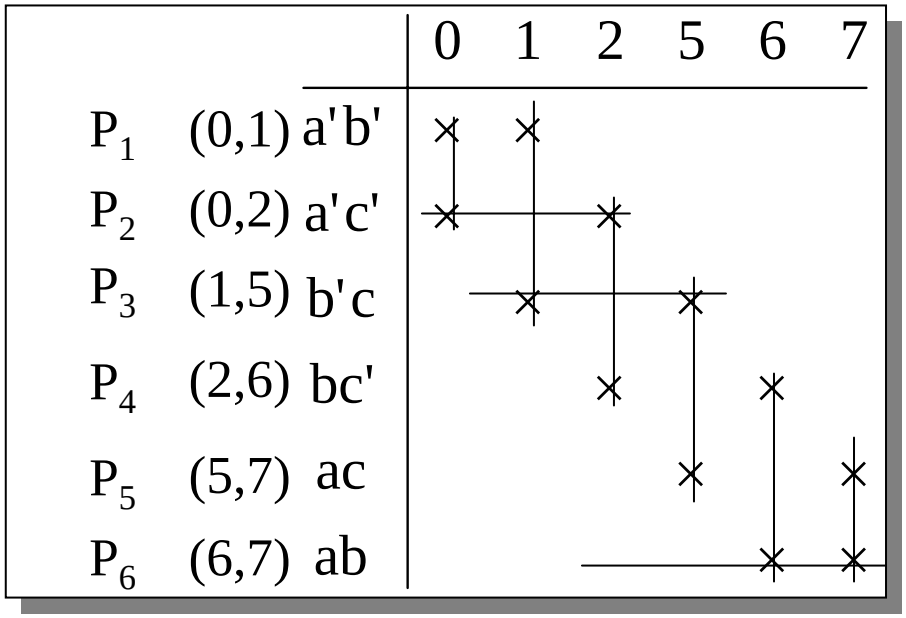
Example $F = \sum m(0,1,2,5,6,7)$

0	0	0	0	∨
1	0	0	1	∨
2	0	1	0	∨
5	1	0	1	∨
6	1	1	0	∨
7	1	1	1	∨

0,1	0	0	–
0,2	0	–	0
1,5	–	0	1
2,6	–	1	0
5,7	1	–	1
6,7	1	1	–



$$\left(\begin{array}{c}\text{All columns have 2 X's} \Rightarrow \text{Use trial and error} \\ F = a'b'+bc'+ac \\ \text{Use another trial and error}\end{array}\right)$$



$$\left(\begin{array}{c}F = a'c'+b'c+ab \\ \Rightarrow 2 \text{ minimum SOP expression !}\end{array}\right)$$

§ Petrick's Method

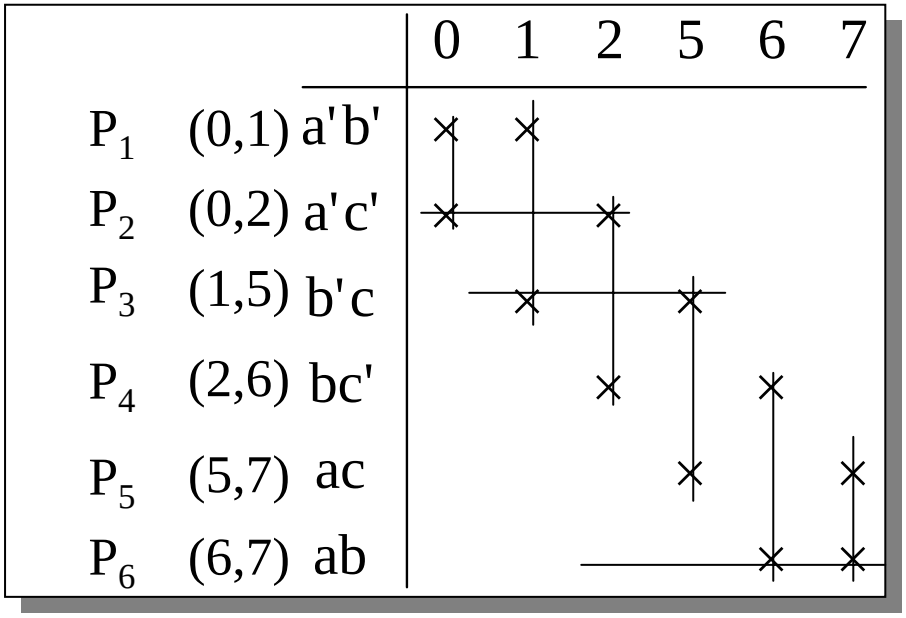
1. Reduce the chart by eliminating the essential prime implicant rows and columns

$P = (P_1 + P_2)(P_1+P_3)(P_2+P_4)(P_3+P_5)(P_4+P_6)(P_5+P_6) = 1$

$= (P_1+ P_2P_3)\ (P_4+P_2P_6)(P_5+P_3P_6)$ //use (X+Y)(X+Z) = X + YZ

$= P_1P_4P_5 + P_1P_2P_5P_6+P_2P_3P_4P_5+P_2P_3P_5P_6+P_1P_3P_4P_6+P_1P_2P_3P_6$
$+P_2P_3P_4P_6+P_2P_3P_6$ //multiply out

$= P_1P_4P_5 + P_1P_2P_5P_6+P_2P_3P_4P_5+P_1P_3P_4P_6+P_2P_3P_6$ //X+XY = X



§ Incompletely Specified Function Simplification

$$F(A,B,C,D) = \sum m(2,3,7,9,11,13) + \sum d(1,10,15)$$

將 d terms 亦放入

d	→	1	0001	∨
		2	0010	∨
		3	0011	∨
		9	1001	∨
d	→	10	1010	∨
		7	0111	∨
		11	1011	∨
		13	1101	∨
d	→	15	1111	∨

(1,3)	00−1	∨
(1,9)	−001	∨
(2,3)	001−	∨
(2,10)	−010	∨
(3,7)	0−11	∨
(3,11)	−011	∨
(9,11)	10−1	∨
(9,13)	1−01	∨
(10,11)	101−	∨
(7,15)	−111	∨
(11,15)	1−11	∨
(13,15)	11−1	∨

(1,3,9,11)	−0−1	∨
(2,3,10,11)	−01−	∨
(3,7,11,15)	−− 11	∨
(9,11,13,15)	1−− 1	∨

No d terms

	2	3	7	9	11	13
(1,3,9,10)		×		×	×	
(2,3,10,11)	×	×			×	
(3,7,11,15)		×	×		×	
(9,11,13,15)				×	×	×

$$F = B'C + CD + AD$$

d terms 10, 15 have been assigned 1

§Simplification Using Map – Entered Variables

↖ ∵ Q – M method not efficient for functions with many variables but few terms

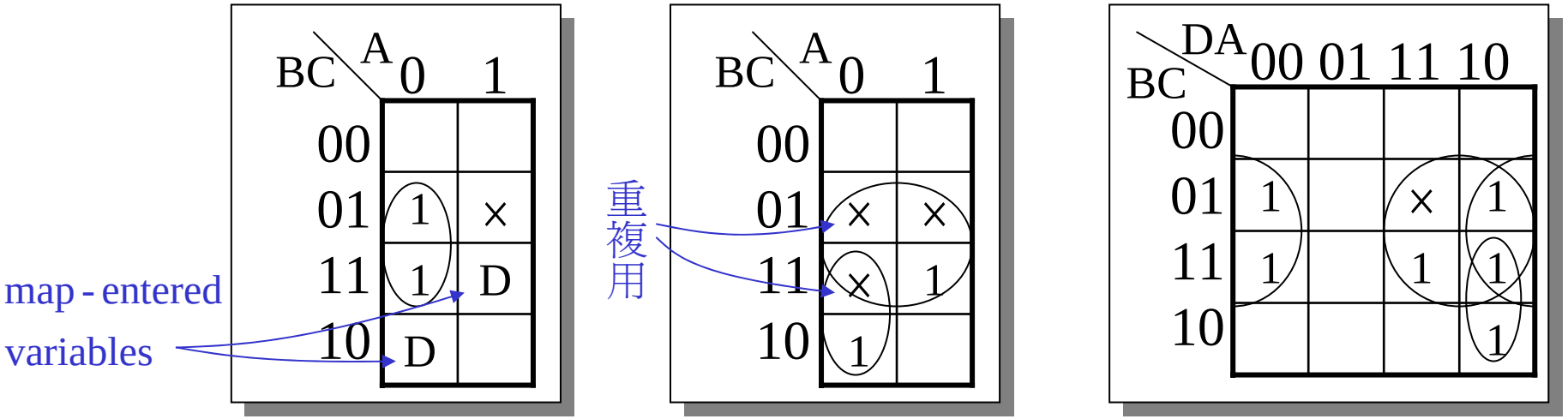
Example

$$F(A,B,C,D) = A'B'C + A'BC + A'BC'D + ABCD + \overset{d}{(AB'C)}$$

$$= \underbrace{A'B'C + A'BC}_{F_0} + \underbrace{(A'BC' + ABC)}_{F_1}D + \overset{d}{(AB'C)}$$

$$= F_0 + F_1 D$$

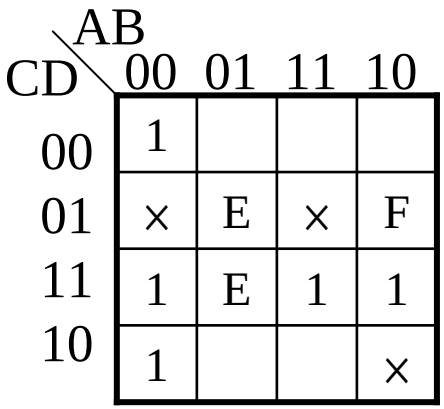
用比較小的 *K – map* 來代表比較多 variables 的 function



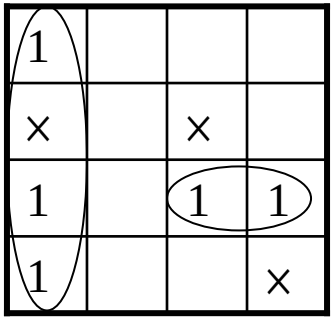
$D = 0 \quad F_0 = A'C$ ， $D = 1 \quad F_1 = C + A'B$

$$\Rightarrow F = F_0 + F_1 D = A'C + D(C + A'B) = A'C + CD + A'BD_{10}$$

Example $G(A,B,C,D,E,F) = m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15} + \sum d(1,10,13)$

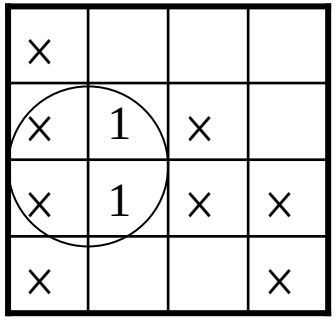


G



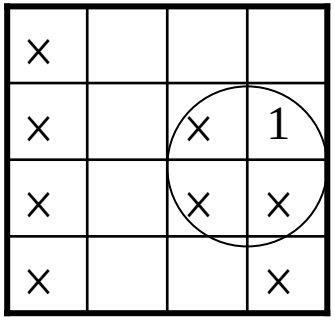
$E = 0, F = 0$

$MS_0 = A'B'+ACD$



$E = 1, F = 0$

$MS_1 = A'D$



$E = 0, F = 1$

$MS_2 = AD$

$$\therefore G = MS_0 + (E)MS_1 + (F)MS_2$$
$$= A'B'+ACD + A'DE + ADF$$

用size比較小的K－map而得variable較多之化簡

General form : $F = MS_0 + P_1 MS_1 + P_2 MS_2 + \cdots\cdots$

MS_0 : All $Pi = 0$;

MS_1 : All $Pi = 0$ except P_1;

but MS_0 don't care

MS_2 : All $Pi = 0$ except P_2;

but MS_0 don't care

MS_3 : All $Pi = 0$ except P_3;

but MS_0 don't care

$\vdots$

$\vdots$

Homework of Chap. 6

9 . (c)

12 . (a) , (b)

15 .

20 .