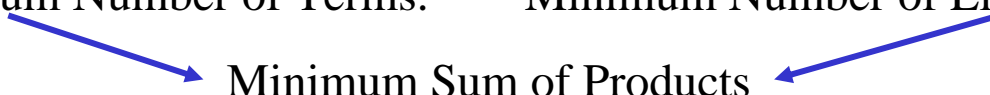
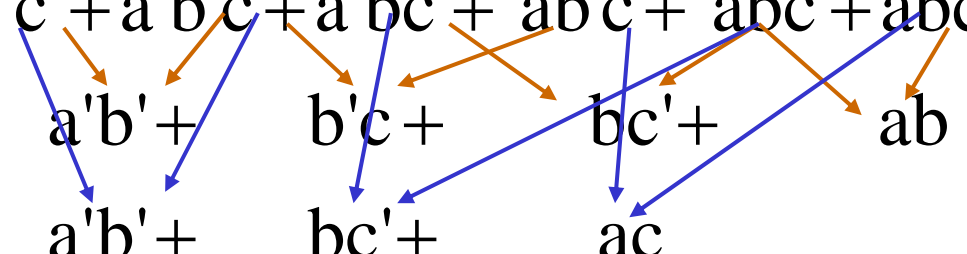


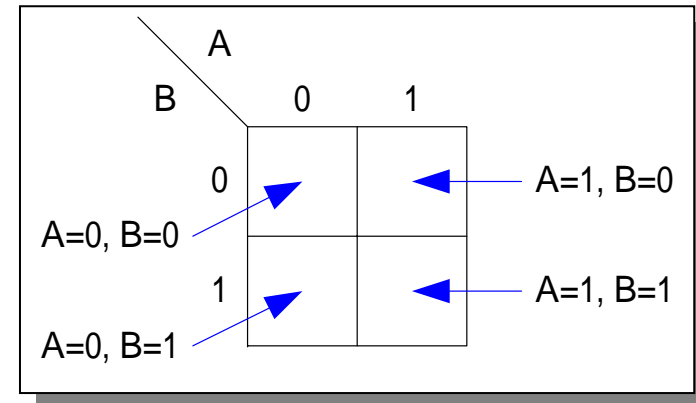
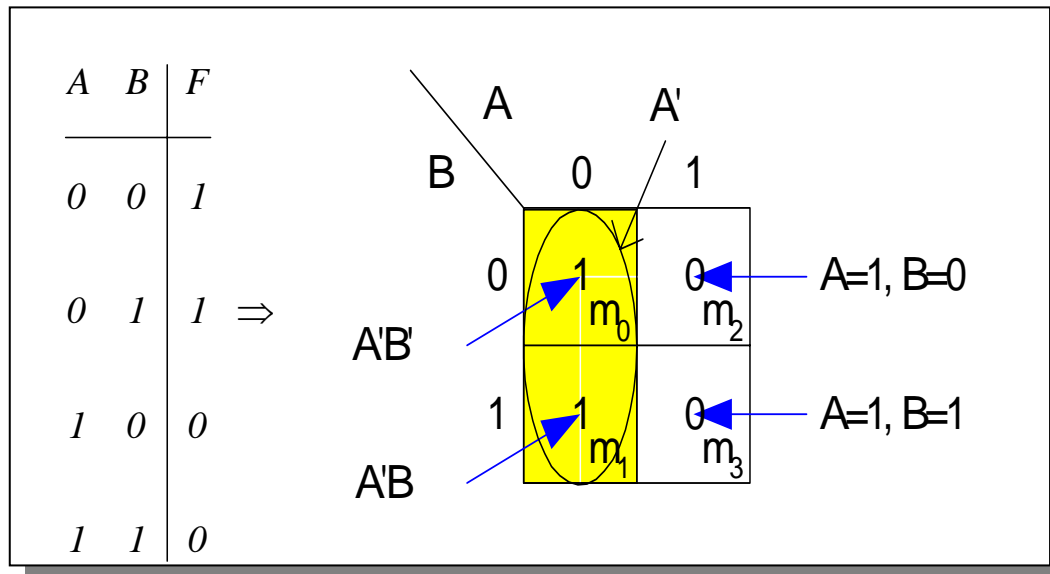
Chap 5. Karnaugh Maps

A method to simplify Boolean function: **Faster, simpler & optimum solution.**

Minimum Number of Terms. Minimum Number of Literals.

 Minimum Sum of Products

$$\begin{aligned}
 F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc = F_m(0,1,2,5,6,7) \\
 &= a'b' + b'c + bc' + ab \\
 &= a'b' + bc' + ac
 \end{aligned}$$


§2-3-variable Karnaugh Maps

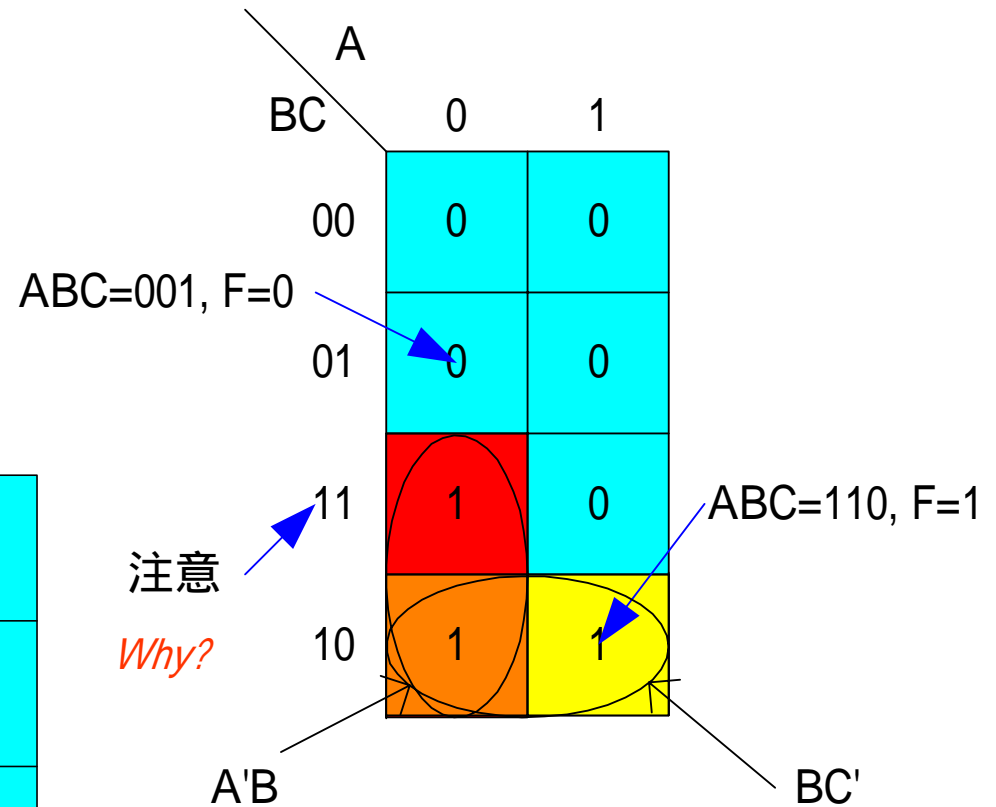


$$F = A'B' + A'B = A'(B + B') = A' \quad \leftarrow \text{uniting theorem}$$

Example 1:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

a	bc	0	1
00	m_0 000	m_4 100	
01	m_1 001	m_5 101	
11	m_3 011	m_7 111	
10	m_2 010	m_6 110	



$$\begin{aligned}
 F &= \sum m(2, 3, 6) \\
 &= A'BC' + A'BC + ABC' \\
 &= A'B + BC'
 \end{aligned}$$

Example 2:

	a		
	bc	0	1
00		m_0	m_4
01		m_1	m_5
11		m_3	m_7
10		m_2	m_6

	a		
	bc	0	1
00		0	0
01		1	1
11		1	0
10		0	0

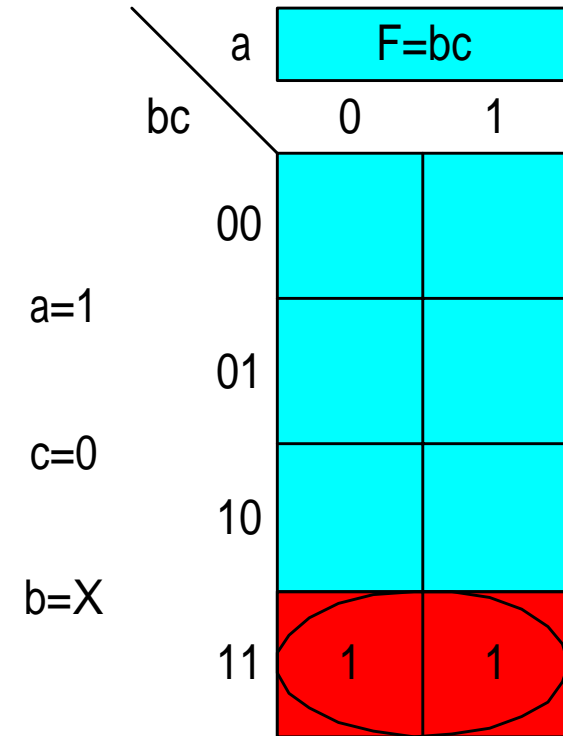
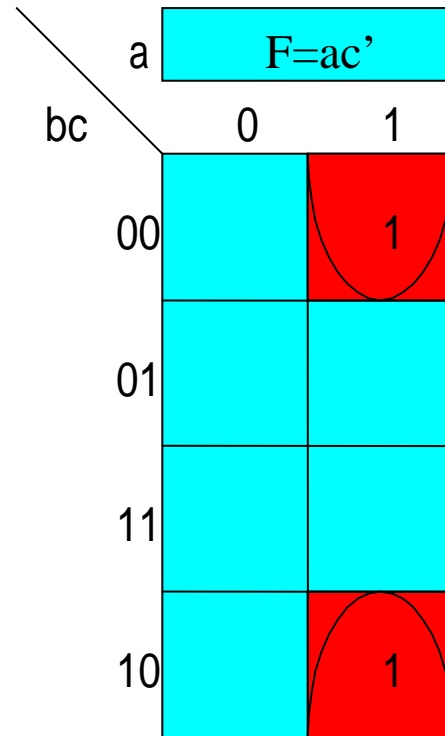
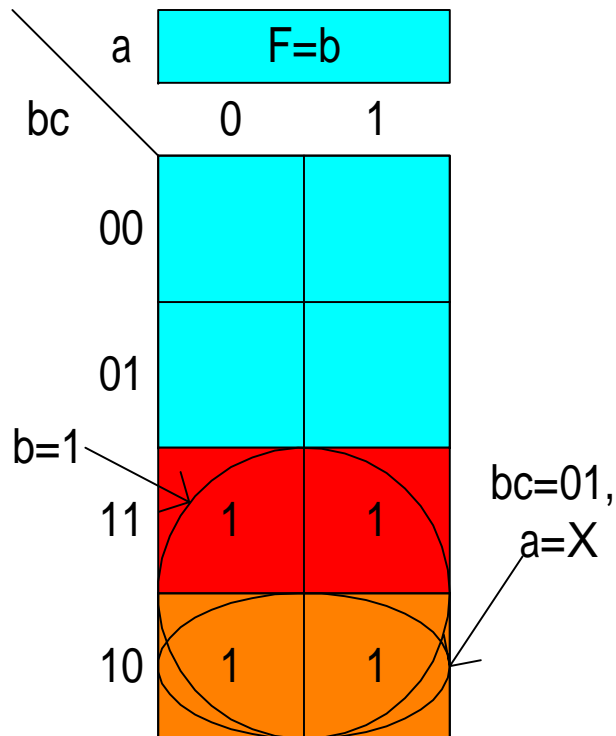
The Karnaugh map shows the function F with minterms 1, 3, and 5 highlighted.
 - Minterm 1 (orange) and minterm 5 (yellow) are grouped together by a circle labeled $b'c$.
 - Minterm 1 (orange) and minterm 3 (red) are grouped together by a circle labeled $a'c$.

$$\begin{aligned}
 F &= \sum m(1, 3, 5) \\
 &= \prod M(0, 2, 4, 6, 7) \\
 &= a'b'c + a'bc + ab'c \\
 &= a'c + b'c
 \end{aligned}$$

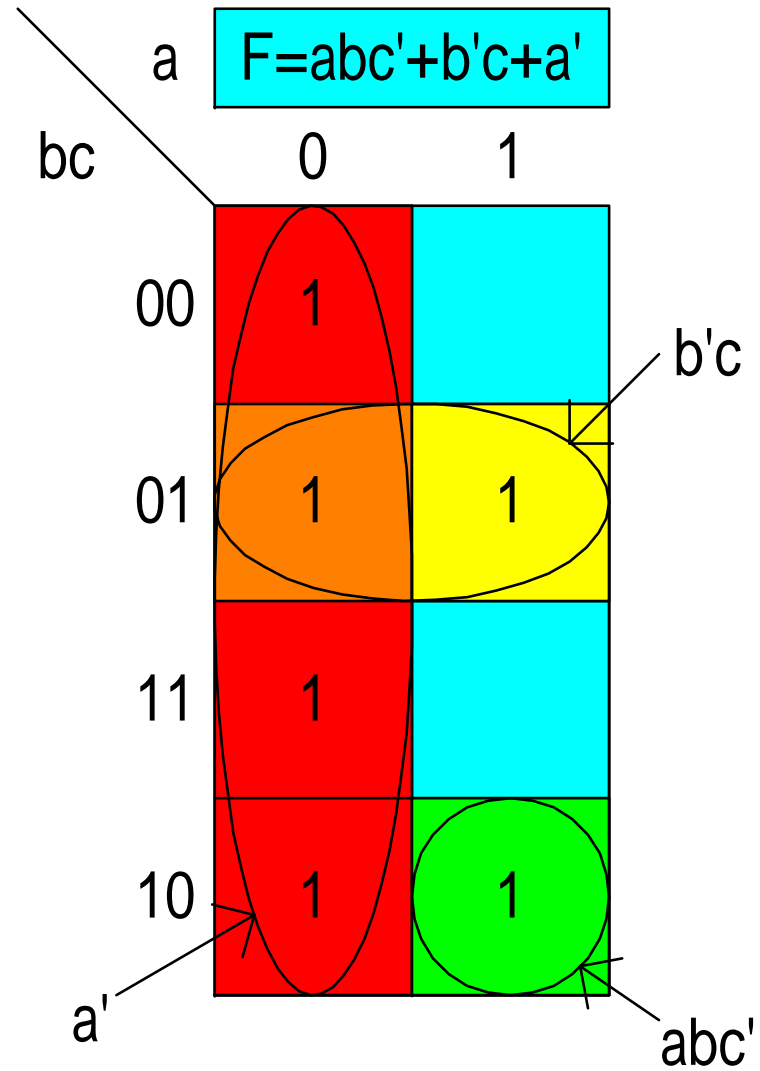
Simplification by Karnaugh Maps

Adjacency between each neighboring minterm

How minterms are plotted:

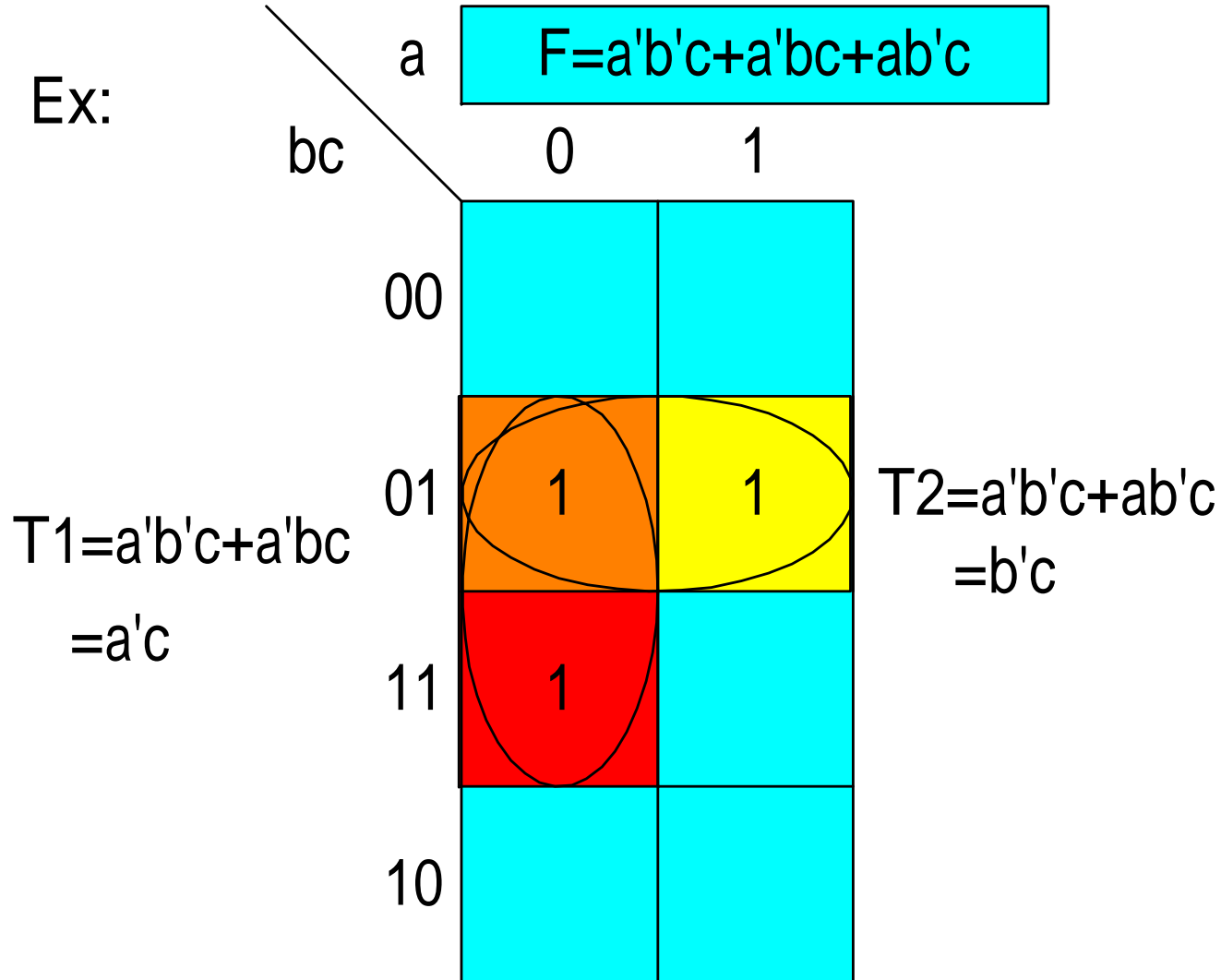


Karnaugh Plot directly from algebraic form

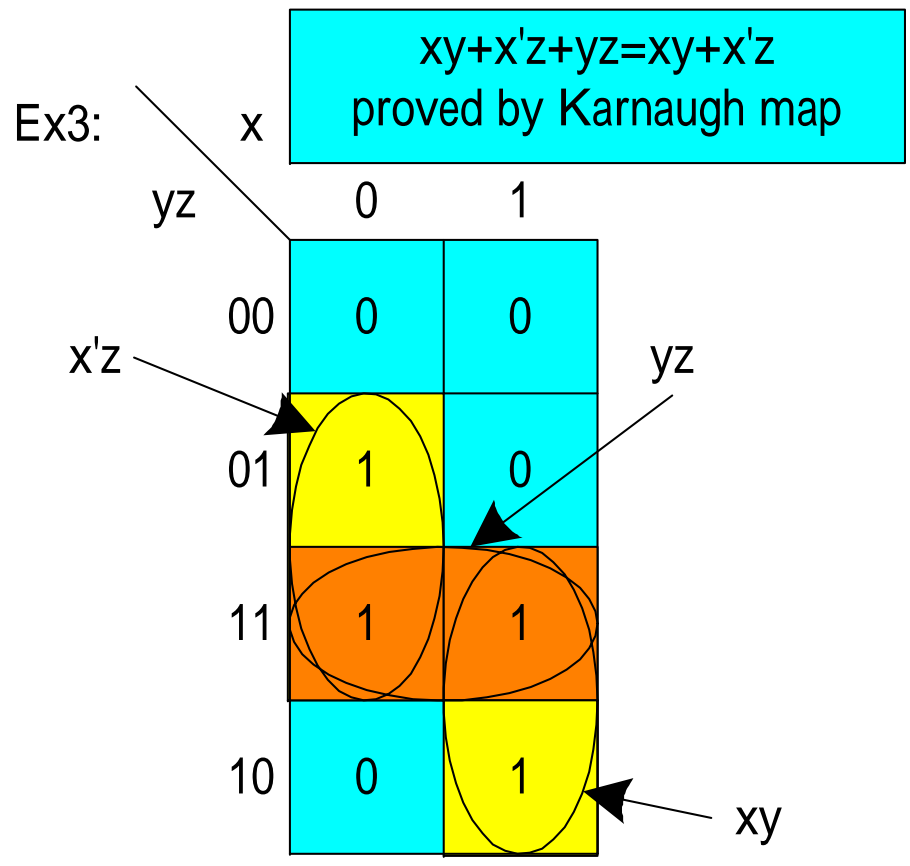
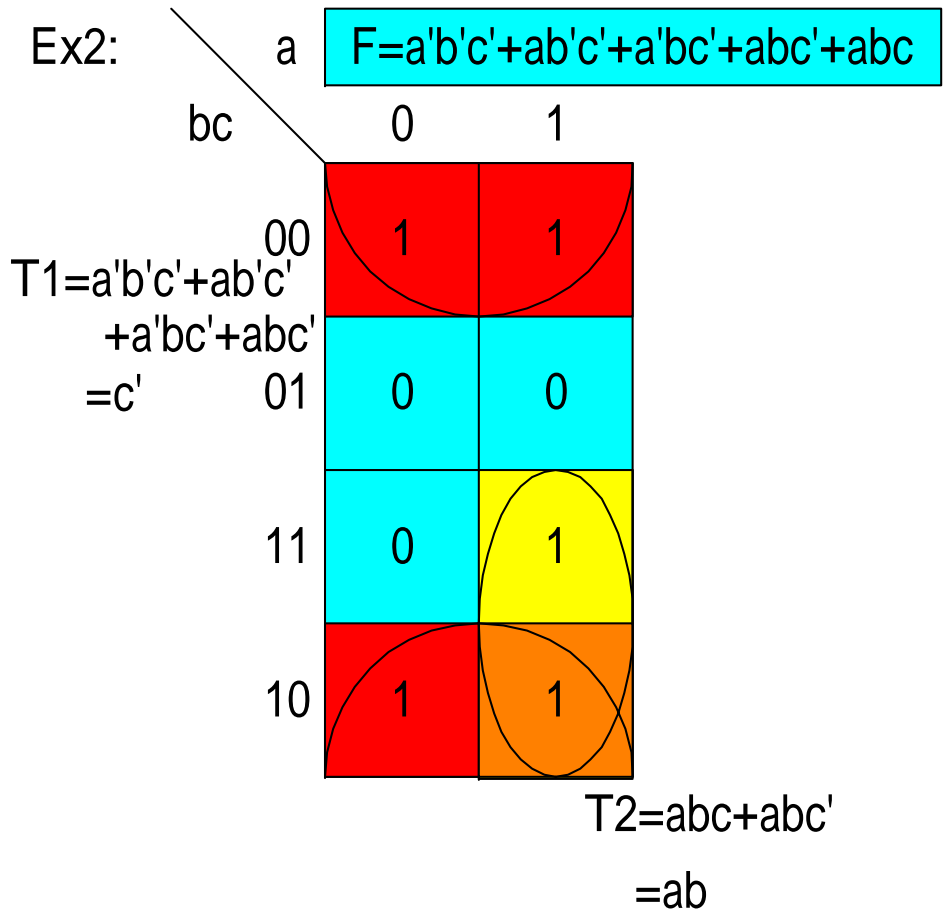


Simplification Using Uniting Theorem: $X \cdot (Y+Y')=X$

Ex:



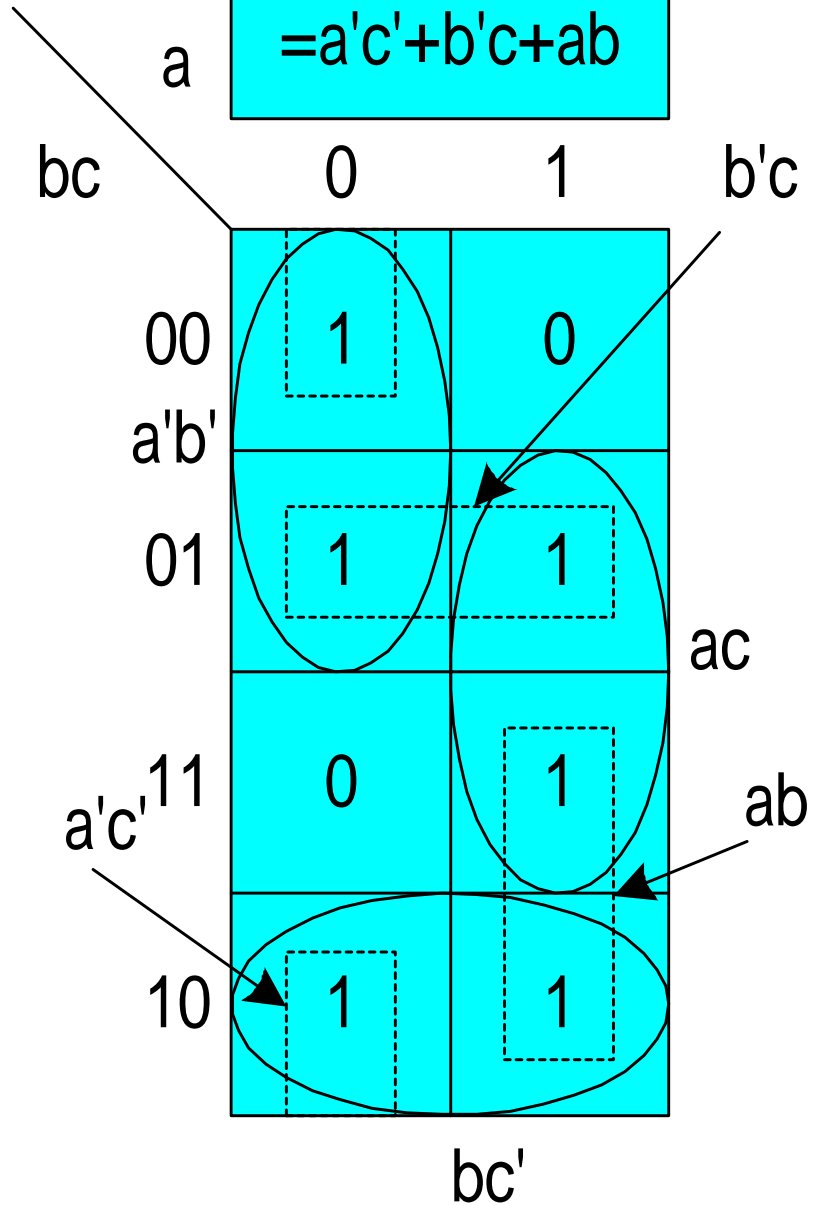
$a'c$ covers minterms: $(1, 3) = a'b'c + a'bc$



Ex4:

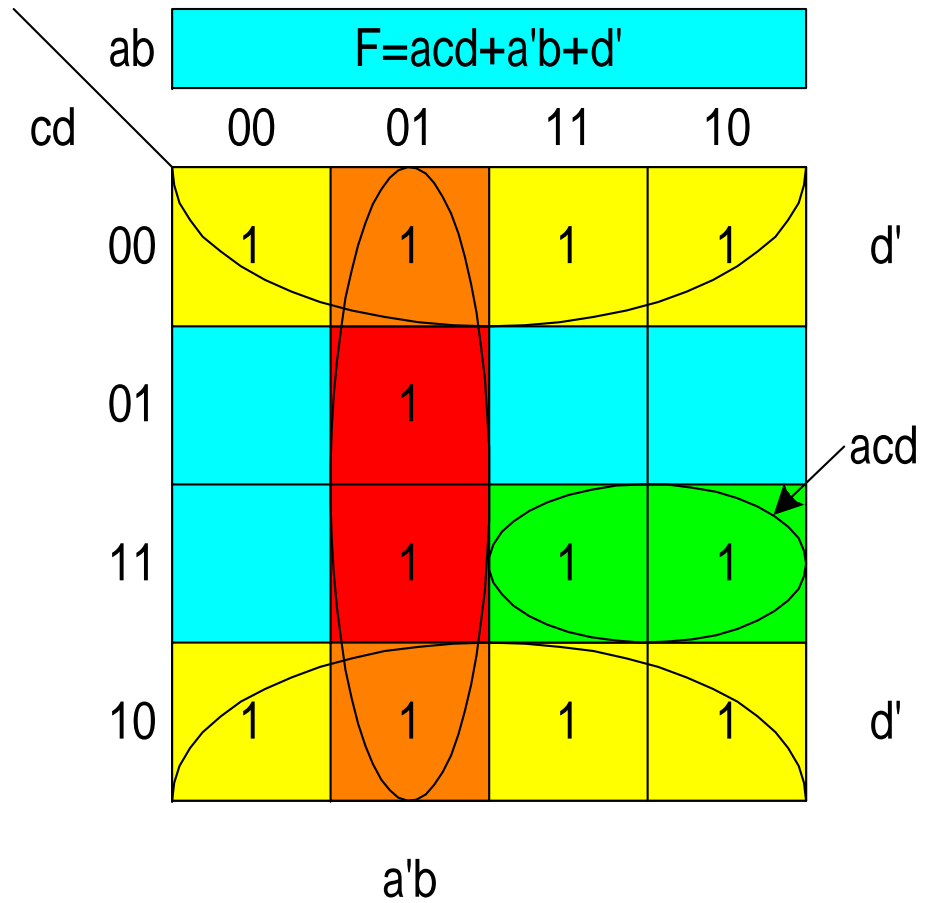
$$F = a'b' + bc' + ac$$
$$= a'c' + b'c + ab$$

Two Minimum forms of the same function



4-Variable Karnaugh Maps

	ab	00	01	11	10
cd	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

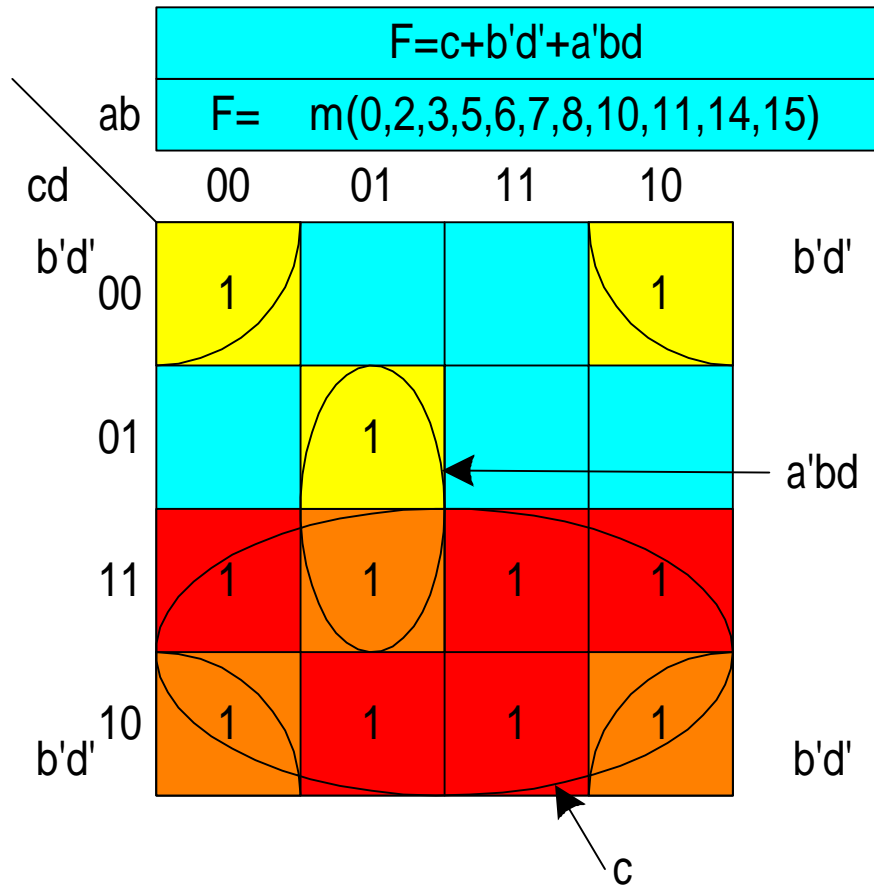


Example1: Simplification

		ab		bc'		
	cd	00	01	11	10	
00		0	1	1	0	
01		1	1	1	0	
a'b'd		1	0	0	0	
11		0	0	0	1	ab'cd'
10		0	0	0	1	

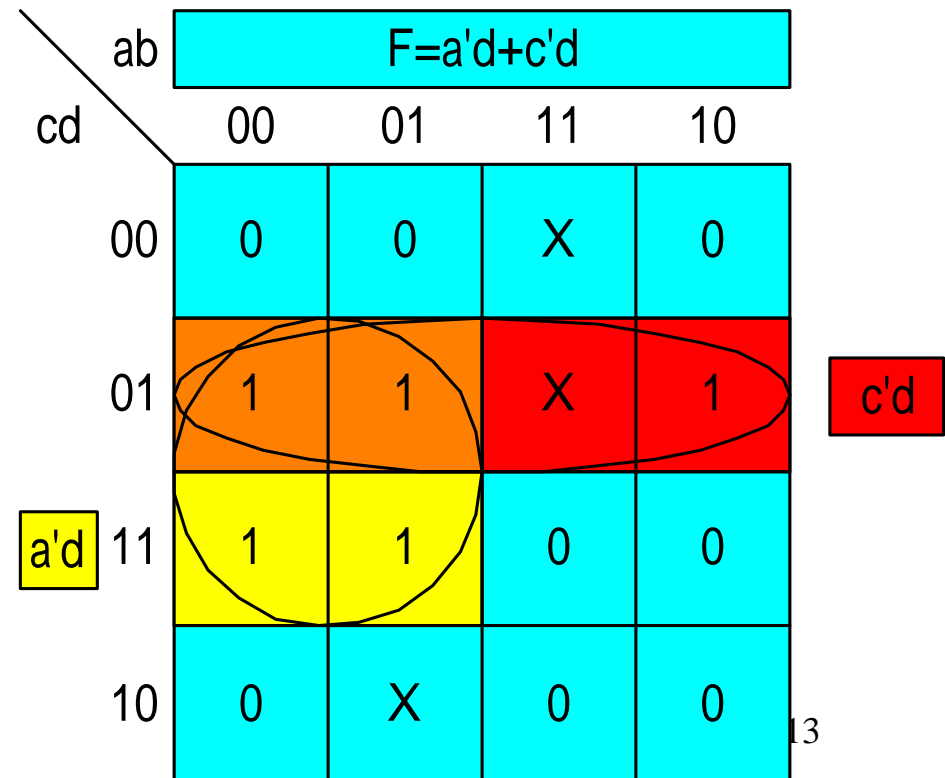
$F(a,b,c,d) = (1,3,4,5,10,12,13)$
 $F = bc' + a'b'd + ab'cd'$

Example 2:

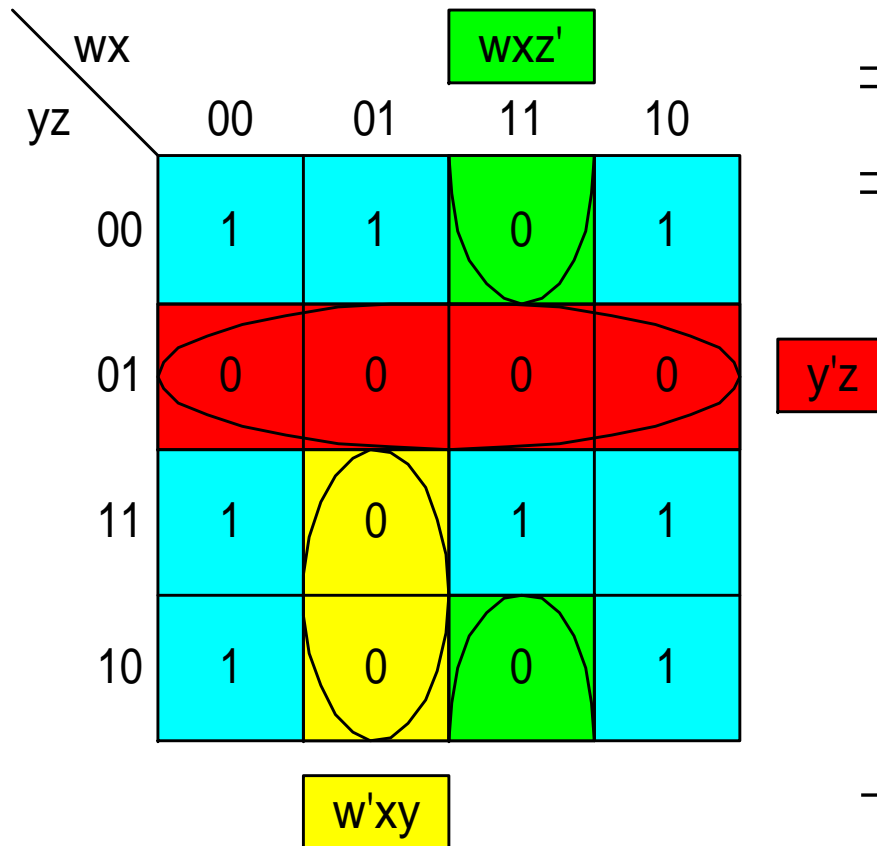


With Don't Care X

$$F = \sum m(1,3,5,7,9) + \sum d(6,12,13)$$



To Obtain **Minimum POS form** from Karnaugh Map



$$F = \sum m(0,2,3,4,8,10,11,15)$$

$$\Rightarrow F' = \sum m(1,5,6,7,9,12,13,14)$$

$$\Rightarrow \underbrace{(F')}'_F = \prod m(1,5,6,7,9,12,13,14)$$

$$F' = y' z + wxz' + w' xy$$

$$\rightarrow F = (y + z')(w' + x' + z)(w + x' + y)$$

補充

Implicant : Single element of the ON - SET or any group of elements that can be combined together in a K - Map.

Prime Implicant : Implicant that can not be combined with another implicant to eliminate a term.

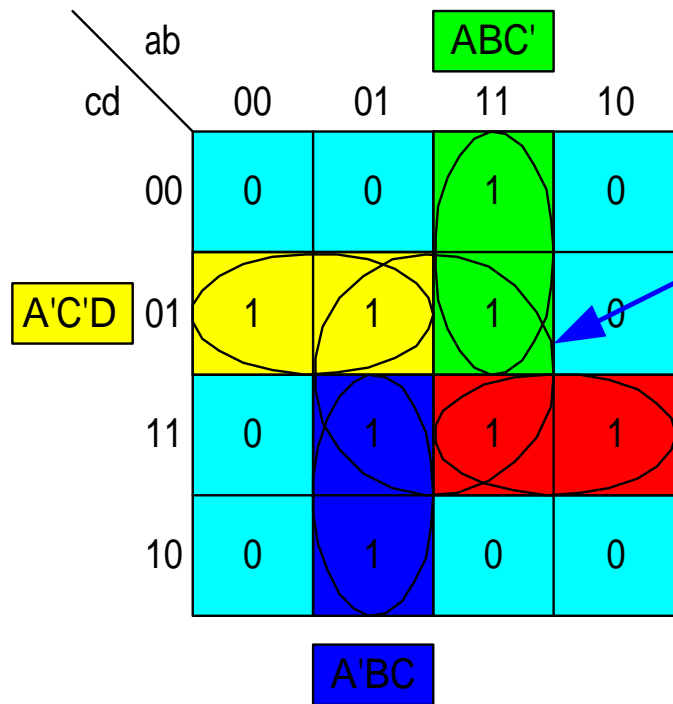
Essential Prime Implicant : if an element of the ON - SET is covered by a single prime implicant, the prime implicant is an essential prime implicant.

Goal:

Grow implicants into **prime implicants**. Cover the ON-SET with as **few** prime implicants as possible

Essential Prime participate in **ALL** possible Covers.

Ex1 :

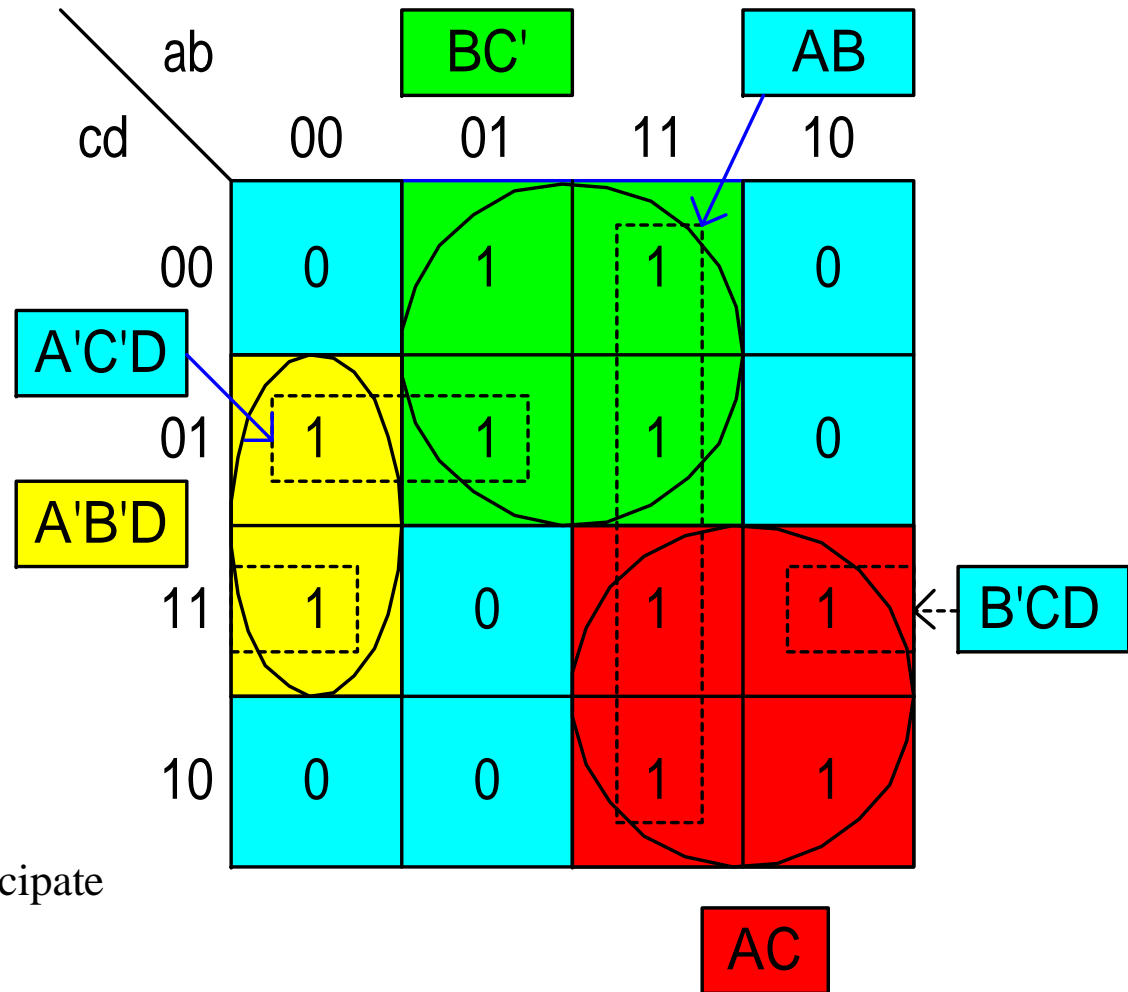


m_1, m_5 m_{12}, m_{13} m_6, m_7 m_{11}, m_{15}
 Implicants $A'C'D$ ABC' $A'BC$ ABC BD
 Prime implicants $AC'D$ ABC' $A'BC$ ABC BD
 Essential Prime Implicants $AC'D$ ABC' $A'BC$ ABC

$$F = \underbrace{ABC' + ACD + A'BC + A'C'D}_{\text{Minimum Cover}} + BD$$

Prime Implicants : $A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

Essential Implicants : BC' , AC

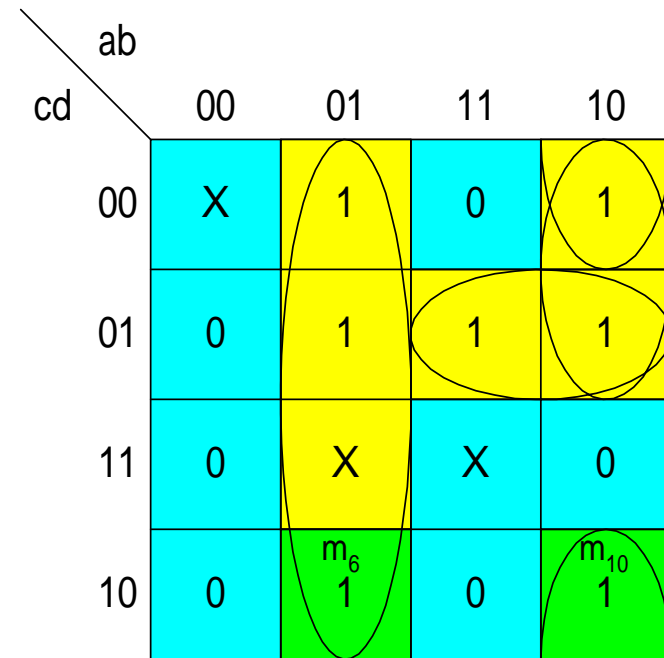
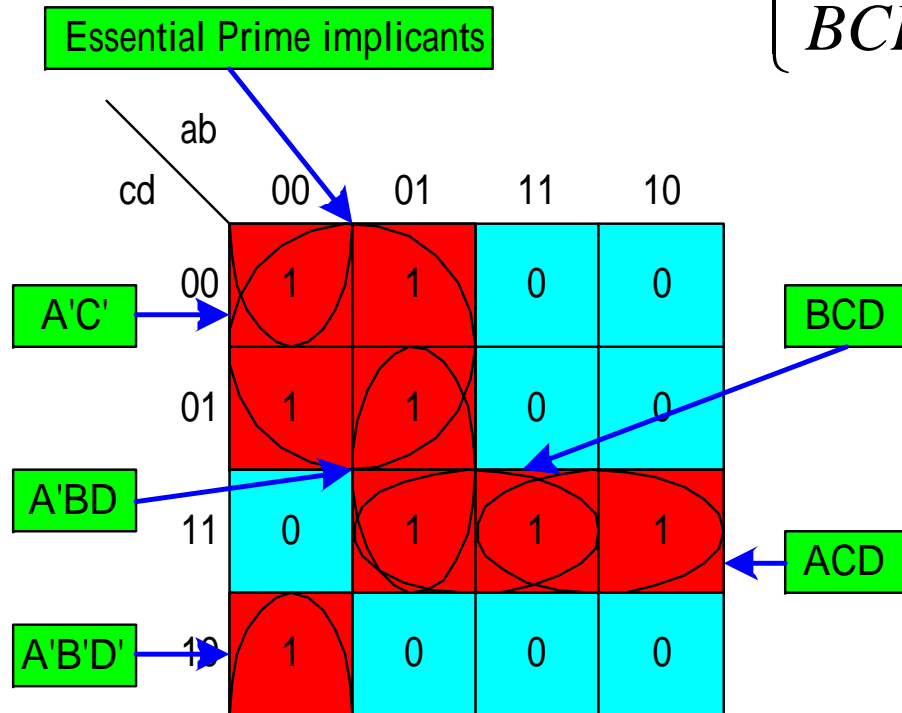


Essential Prime Implicants participate in all possible covers

$$F = \overbrace{BC' + AC} + A'C'D + B'CD$$

$$F = BC' + AC + A'B'D \leftarrow \text{Minimum Cover}$$

$$F = A' C' + A' B' D' + A C D + \left\{ \begin{array}{l} A' B D \\ \text{or} \\ B C D \end{array} \right.$$



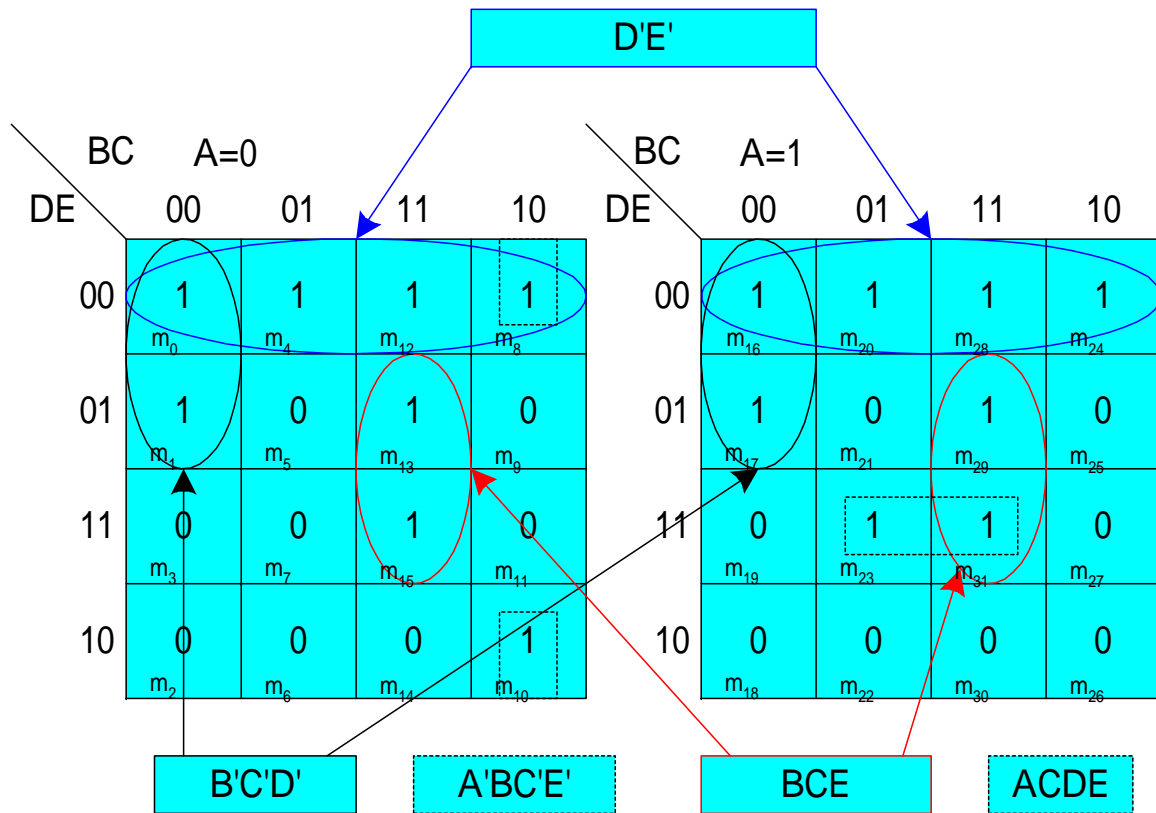
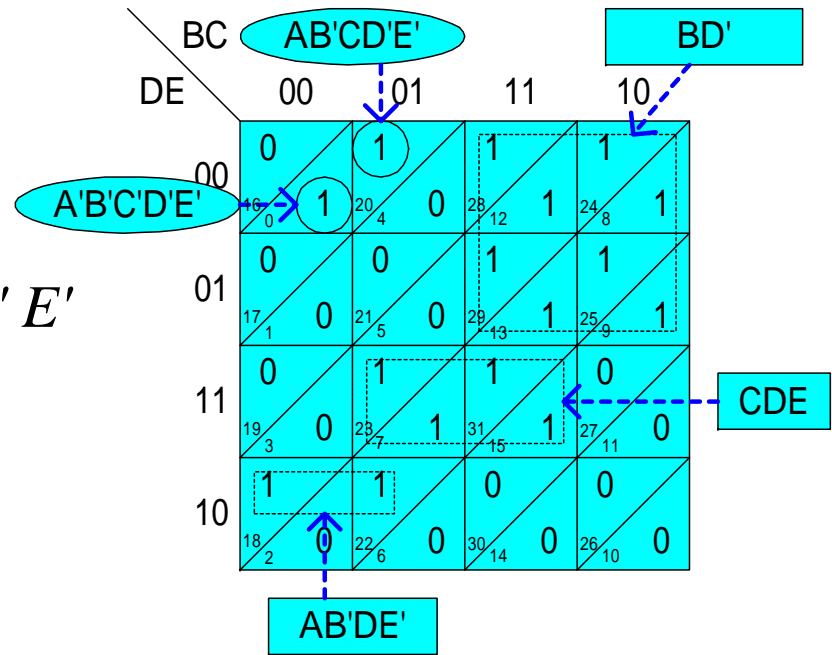
Only $m(6,10)$ are covered by only one prime implicant.

A minimum solution:

must contain all *essential prime implicants*

5 – Variable Karnaugh Maps

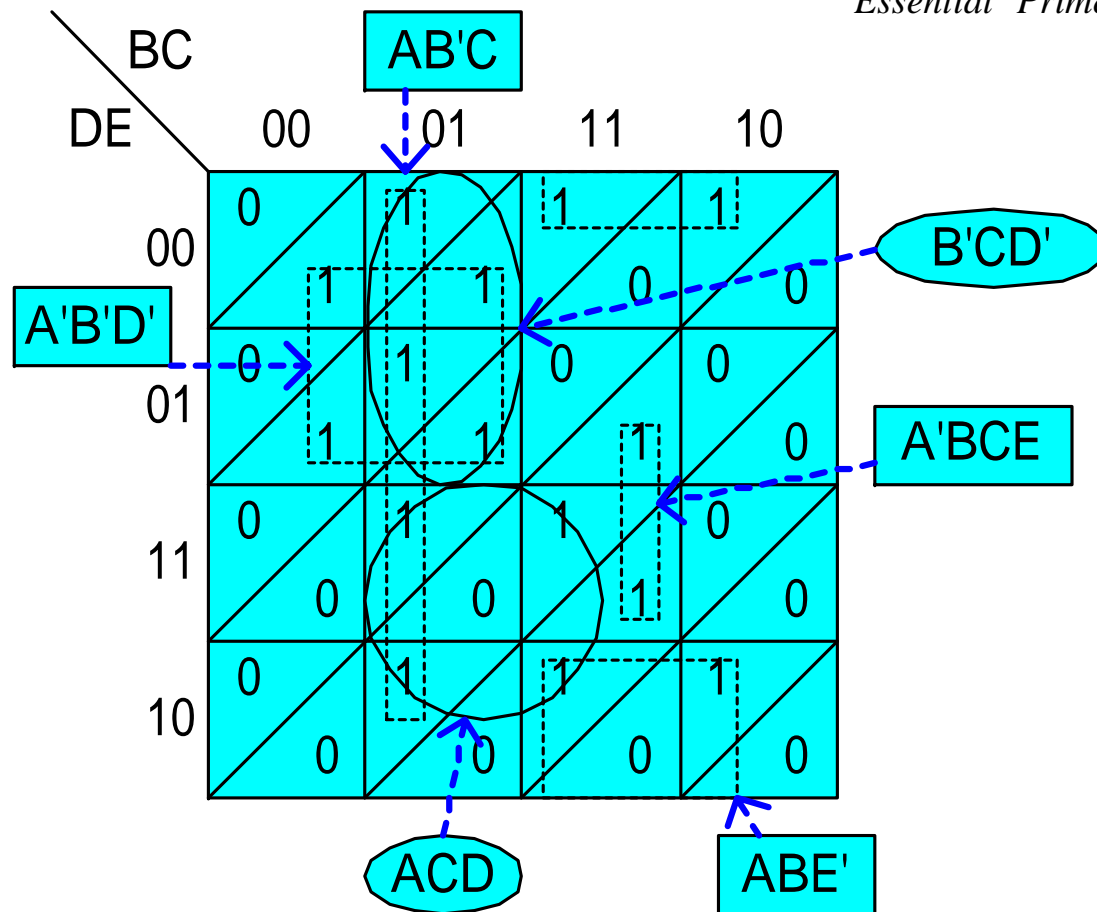
$$F = A' B' C' D' E' + A' B' C D' E' + AB' DE' + CDE + BD'$$



$$F = D' E' + B' C' D' + BCE + A' B C' E' + ACDE$$

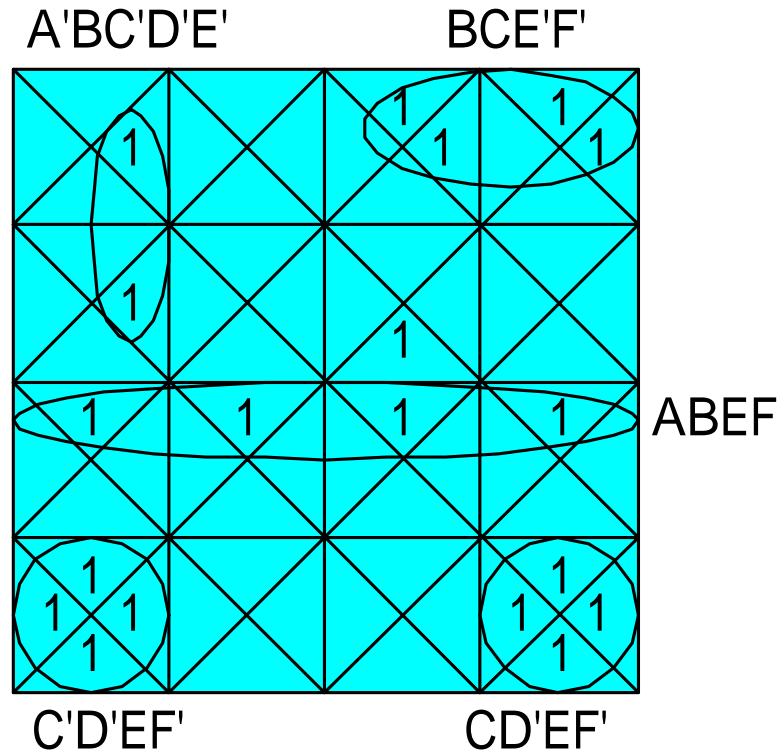
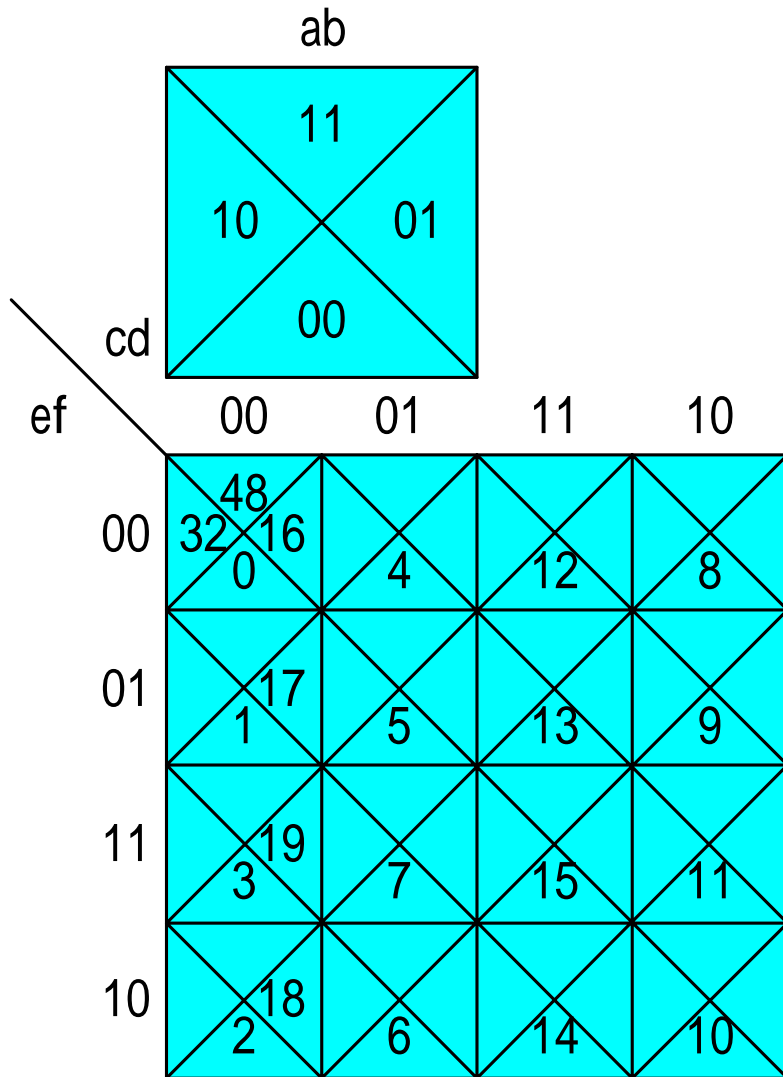
Another Example

$$F = \underbrace{A'B'D + ABE' + ACD + A'BCE}_{\text{Essential Prime Implicants}} + \left\{ \begin{array}{l} AB'C \\ \text{or} \\ B'CD' \end{array} \right.$$



$$F(A, B, C, D, E) = \sum m(0, 1, 4, 5, 13, 15, 20, 21, 22, 23, 24, 26, 28, 30, 31)$$

6-Variable K-Maps



Homework Chap 5.

- 1:
- 2:
- 4:
- 5:
- 7: (b) (d)

- 10: (b)
- 12:
- 19:
- 21: (b)
- 27: