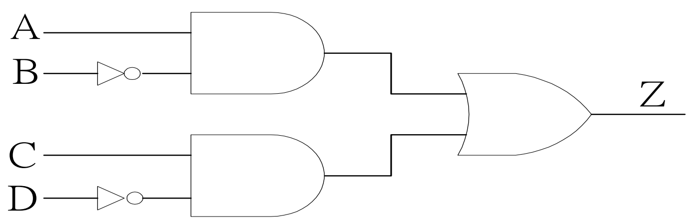
Unit 4. App., Minterm, Maxterm of Boolean Algebra

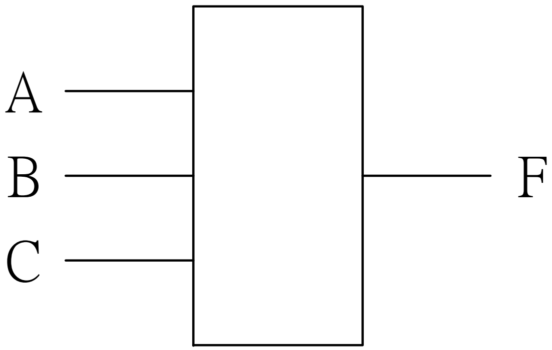
Boolean Equation Formation from Sentence

$$Ex: \underbrace{The\ alarm\ will\ ring}_{Z}\quad iff\quad \underbrace{the\ power\ of\ alarm\ is\ on}_{A}\quad and$$
$$\underbrace{the\ door\ is\ not\ closed}_{B'}\quad or\quad \underbrace{it\ is\ after\ 6\,p.m.}_{C}\quad and$$
$$\underbrace{the\ window\ is\ not\ closed}_{D'}$$

$$Z = AB' + CD'$$



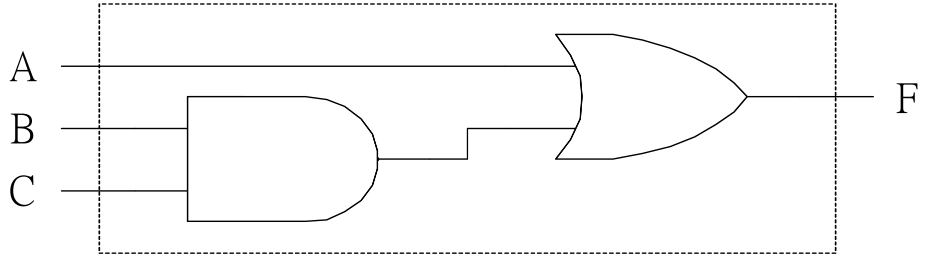
§ Combinational Network Design Using a Truth Table



A	B	C	f	f'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

① By “1's” function

$$f = A'BC + AB'C' + AB'C + ABC' + ABC$$
$$= A'BC + AB' + AB$$
$$= A'BC + A$$
$$= A + BC$$



② By "0's" function

$$f=(A+B+C)(A+B+C')(A+B'+C)$$

$$=(A+B)(A+B'+C)=A+B(B'+C)$$

$$=A+BC$$

f *is* "0" ,

if $A=B=C=0$

$A=B=0, C=1$

$A=0=C, B=1$

A	B	C	f	$f\ '$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

③ By $f\ '$

$$f'=A'B'C'+A'B'C+A'BC'$$

$$\Downarrow$$

$$f=(A+B+C)(A+B+C')(A+B'+C)$$

§ Minterm & Maxterm

$Variables \; : X, Y, Z \cdots A, B, C \cdots$

$Literals \; : X, X', Y, Y' \cdots\cdots$

Example

$F = ABC' + A'B' + BC'$

3 variables 7 literals

Minterm and Maxterm

RowNo.	*A*	*B*	*C*	*Mintems*	*Maxterms*	*f*	*f'*
0	*0*	*0*	*0*	$A'B'C' = m_0$	$A+B+C = M_0$	*0*	*1*
1	*0*	*0*	*1*	$A'B'C = m_1$	$A+B+C' = M_1$	*0*	*1*
2	*0*	*1*	*0*	$A'BC' = m_2$	$A+B'+C = M_2$	*0*	*1*
3	*0*	*1*	*1*	$A'BC = m_3$	$A+B'+C' = M_3$	*1*	*0*
4	*1*	*0*	*0*	$AB'C' = m_4$	$A'+B+C = M_4$	*1*	*0*
5	*1*	*0*	*1*	$AB'C = m_5$	$A'+B+C' = M_5$	*1*	*0*
6	*1*	*1*	*0*	$ABC' = m_6$	$A'+B'+C = M_6$	*1*	*0*
7	*1*	*1*	*1*	$ABC = m_7$	$A'+B'+C' = M_7$	*1*	*0*

Canonical form $m_i' = M_i$

$$\Rightarrow \quad f = A'BC + AB'C' + AB'C + ABC' + ABC$$

$$= m_3 + m_4 + m_5 + m_6 + m_7$$

$$or \quad f(A, B, C) = \sum m(3,4,5,6,7)$$

任何 minterm $= 1 \Rightarrow f = 1$

$$f = (A + B + C)(A + B + C')(A + B' + C) = M_0 M_1 M_2$$

$$f(A, B, C) = \prod M(0, 1, 2)$$

任何 $maxterm = 0 \Rightarrow f = 0$

表示法：

$$f(A,B,C)=\sum m(3,4,5,6,7) \Rightarrow f_A'(A,B,C)=m_0+m_1+m_2$$

$$(f')'=f=(m_0+m_1+m_2)'=m_0'\cdot m_1'\cdot m_2'=M_0M_1M_2$$

$$=\Pi M(0,1,2)$$

又 $f'=(m_3+m_4+m_5+m_6+m_7)'=m_3'\cdot m_4'\cdot m_5'\cdot m_6'\cdot m_7'$

$$=M_3M_4M_5M_6M_7=\Pi M(3,4,5,6,7)$$

Another Example

A	B	C	D	g	g'	
0	0	0	0	0	1	m_0
0	0	0	1	0	1	m_1
0	0	1	0	1	0	m_2
0	0	1	1	1	0	m_3
0	1	0	0	1	0	m_4
0	1	0	1	0	1	m_5
0	1	1	0	1	0	m_6
0	1	1	1	1	0	m_7
1	0	0	0	0	1	m_8
1	0	0	1	0	1	m_9
1	0	1	0	1	0	m_{10}
1	0	1	1	1	0	m_{11}
1	1	0	0	0	1	m_{12}
1	1	0	1	0	1	m_{13}
1	1	1	0	1	0	m_{14}
1	1	1	1	0	1	m_{15}

$$g = \sum m(2,3,4,6,7,10,11,14)$$
$$\text{or} = \prod M(0,1,5,8,9,12,13,15)$$
$$g' = \sum m(0,1,5,8,9,12,13,15)$$
$$g = \left[\sum m(0,1,5,8,9,12,13,15)\right]'$$
$$= \prod m_i'(0,1,5,8,9,12,13,15)$$
$$= \prod M_i(0,1,5,8,9,12,13,15)$$

Ex：1. Find "Minterm Expansion" of $f = a'(b' + d) + acd'$

$$f = a'b' + a'd + acd'$$
$$= a'b'(c + c')(d + d') + a'd(b + b')(c + c') + acd'(b + b')$$
$$= a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + a'b'c'd + a'b'cd$$
$$+ a'bc'd + a'bcd + abcd' + ab'cd'$$
$$= \mathbf{0000} + \mathbf{0001} + \mathbf{0010} + \mathbf{0011} + \mathbf{0101} + \mathbf{0111} + \mathbf{1110} + \mathbf{1010}$$
$$= \sum m(\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{5}, \mathbf{7}, \mathbf{10}, \mathbf{14})$$
$$\textit{Find Maxterm expansion } f = \prod M(\mathbf{4}, \mathbf{6}, \mathbf{8}, \mathbf{9}, \mathbf{11}, \mathbf{12}, \mathbf{13}, \mathbf{15})$$

Ex：2. Find "Maxterm Expansion" of $f = a + \overline{b}c$

Ex : 3 Prove : $a'c + b'c' + ab = a'b' + bc + ac'$

$$
\begin{aligned}
\text{左} &= a'c(b+b') + (a+a')b'c' + ab(c+c') \\
&= m_3 + m_1 + m_4 + m_0 + m_7 + m_6 \\
&= \sum m(0,1,3,4,6,7) \\
\text{右} &= a'b'(c+c') + (a+a')bc + ac'(b+b') \\
&= m_1 + m_0 + m_7 + m_3 + m_6 + m_4 \\
&= \sum m(0,1,3,4,6,7)
\end{aligned}
$$

§ General Form for Minterm & Maxterm Expansion

A	B	C	F
0	0	0	a_0
0	0	1	a_1
0	1	0	a_2
0	1	1	a_3
1	0	0	a_4
1	0	1	a_5
1	1	0	a_6
1	1	1	a_7

$$F = a_0 m_0 + a_1 m_1 + a_2 m_2 + \ldots + a_7 m_7 = \sum_{i=0}^{7} a_i m_i \cdots\cdots (a)$$

當 i_{th} term exists $\Rightarrow a_i = 1$

或者：

$$F = (a_0 + M_0)(a_1 + M_1)(a_2 + M_2)\ldots\ldots(a_7 + M_7)$$

$$= \prod_{i=0}^{7} (a_i + M_i) \cdots\cdots (b)$$

當 i_{th} term does not exist $\Rightarrow a_0 = 1$

$\because (1 + M_i) = 1$

A	B	C	F		
0	0	0	0	m_0	M_0
0	0	1	1	m_1	M_1
0	1	0	1	m_2	M_2
0	1	1	0	m_3	M_3
1	0	0	0	m_4	M_4
1	0	1	0	m_5	M_5
1	1	0	1	m_6	M_6
1	1	1	1	m_7	M_7

$$F = 1 \cdot m_1 + 1 \cdot m_2 + 1 \cdot m_6 + 1 \cdot m_7$$
$$= (0 + M_0)(0 + M_3)(0 + M_4)(0 + M_5)$$

由（b）

$$F' = \left[\prod (a_i + M_i)\right]' = \sum (a_i + M_i)'$$

$$= \sum (a_i' \cdot M_i') = \sum (a_i' \cdot m_i)$$

由（a）

$$F' = \left[\sum (a_i \cdot m_i)\right]' = \prod (a_i' + m_i')$$

$$= \prod (a_i' + M_i)$$

於 F 中不存在之 m_i，於 F' 中即存在；

於 F 中不存在之 M_i，於 F' 中即存在。

$$\Rightarrow \text{General Form}: F = \sum_{i=0}^{2^n-1} a_i m_i = \prod_{i=0}^{2^n-1} (a_i + M_i)$$

$$F' = \sum_{i=0}^{2^n-1} a_i' m_i = \prod_{i=0}^{2^n-1} (a_i' + M_i)$$

$\text{Property}: \boxed{m_i m_j = 0 \quad \text{if} \quad i \neq j}$ $\quad \text{Ex}: (ABCD)(AB\overline{C}D) = 0$

$$\text{So}: \ f_1 = \sum_{i=0}^{2^n-1} a_i m_i \qquad f_2 = \sum_{j=0}^{2^n-1} b_j m_j$$

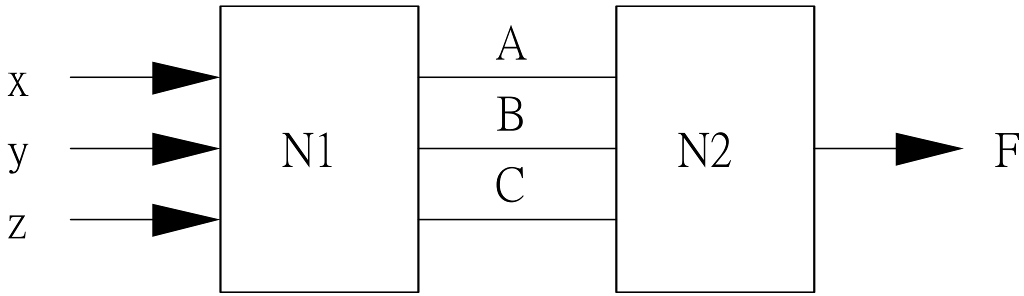
$$f_1 f_2 = \left(\sum_{i=0}^{2^n-1} a_i m_i\right)\left(\sum_{j=0}^{2^n-1} b_j m_j\right) = \sum_{i=0}^{2^n-1} \sum_{j=0}^{2^n-1} a_i b_j m_i m_j$$

$$= \Sigma a_i b_i m_i \quad (i \neq j \quad \text{terms} = 0)$$

$$\text{Ex}: \quad f_1 = \sum m(0,2,3,5,9,11) \quad f_2 = \sum m(0,3,9,11,13,14)$$

$$f_1 f_2 = \sum m(0,3,9,11)$$

§ Incompletely Specified Functions



	x	y	z	A	B	C
	0	0	0	0	0	0
	0	1	1	0	0	0
	0	0	0	0	1	0
N_1	0	1	1	0	1	1
	1	0	0	1	0	0
	1	1	1	1	0	1
	1	0	0	1	0	1
	1	1	1	1	1	1

Ⅰ. x , x = 0 , 0

$$F = A'B'C' + A'BC + ABC$$
$$= A'B'C' + BC$$

Ⅱ. x , x = 1 , 0

$$F = A'B'C' + A'B'C + A'BC + ABC$$
$$= A'B' + BC$$

Ⅲ. x , x = 1 , 1

$$F = A'B'C' + A'B'C + A'BC + ABC' + ABC$$
$$= A'B' + BC + AB$$

	A	B	C	F
	0	0	0	1
	0	0	1	X ← don' t care
	0	1	0	0
N_2	0	1	1	1
	1	0	0	0
	1	0	1	0
	1	1	0	X
	1	1	1	1

Case 2 simplest solution

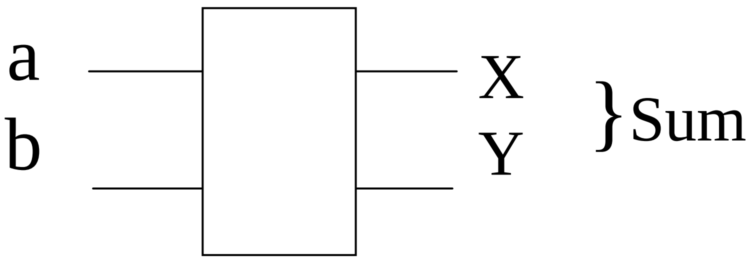
$$F = \sum m(0,3,7) + \sum d(1,6)$$
$$or = \prod M(2,4,5) \cdot \prod D(1,6)$$

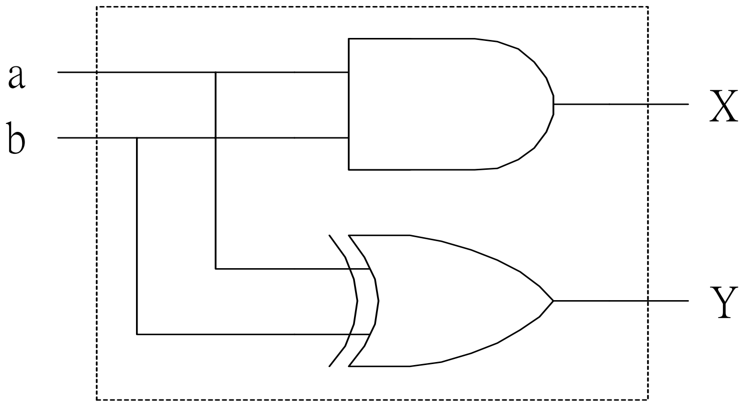
§ Truth Table Construction

Ex 1：Simple Binary Adder

$$X = ab$$

$$Y = ab' + a'b = a \oplus b$$

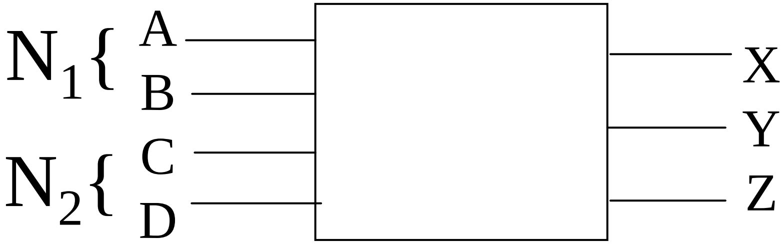




a	b	Sum
0	0	00
0	1	01
1	0	01
1	1	10

X	Y
0	0
0	1
0	1
1	0

Ex 2：Adder Network



$$X(A,B,C,D)=\sum m(7,10,11,13,14,15)$$
$$Y(A,B,C,D)=\sum m(2,3,5,6,8,9,12,15)$$
$$Z(A,B,C,D)=\sum m(1,3,4,6,9,11,12,14)$$

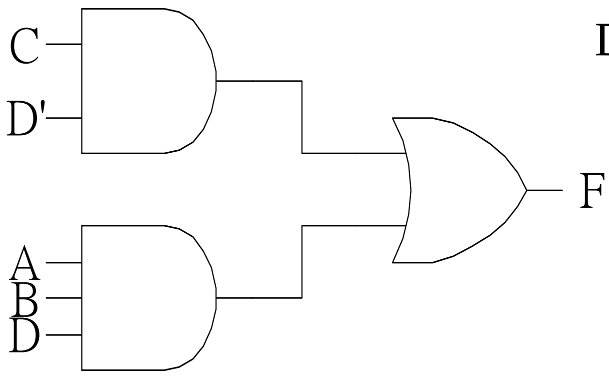
A	*B*	*C*	*D*	*X*	*Y*	*Z*
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	0	1	1
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	0	0
1	0	1	1	1	0	1
1	1	0	0	0	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	1
1	1	1	1	1	1	0

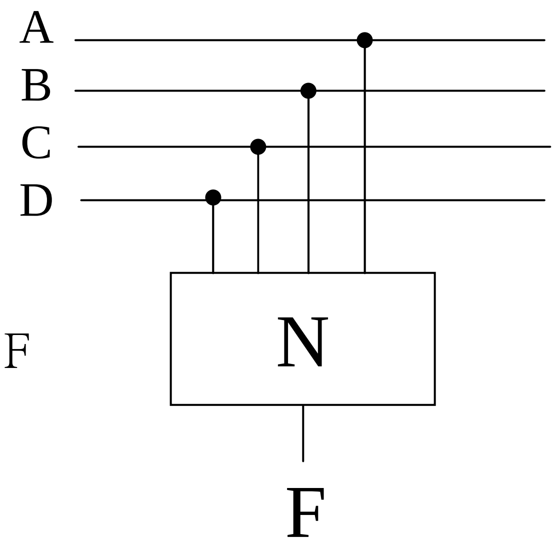
Ex 3：6 – 3 – 1 – 1 BCD Error – Correcting

	A	B	C	D	F
0	0	0	0	0	0
1	0	0	0	1	0
1	0	0	1	0	1
2	0	0	1	1	0
3	0	1	0	0	0
4	0	1	0	1	0
4	0	1	1	0	1
5	0	1	1	1	0
6	1	0	0	0	0
7	1	0	0	1	0
7	1	0	1	0	1
8	1	0	1	1	0
9	1	1	0	0	0
10	1	1	0	1	1
10	1	1	1	0	1
11	1	1	1	1	1

F gives 1 if not correct

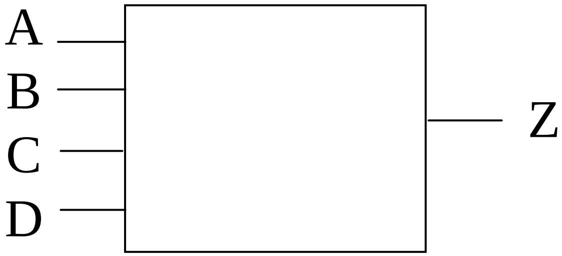
$$
\begin{aligned}
F &= \sum m(2, 6, 10, 13, 14, 15) \\
&= A'B'CD' + A'BCD' + AB'CD' + ABCD' \\
&\quad + ABC'D + ABCD \\
&= A'CD' + ACD' + ABD \\
&= CD' + ABD
\end{aligned}
$$





Ex 4：Input 8421 BCD

if divisible by 3 , output Z=1 ,
otherwise Z=0



$$Z = \sum m(0,3,6,9) + \sum d(10,11,12,13,14,15)$$

A	*B*	*C*	*D*	*Z*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Homework

1 , 2 , 5 , 7 , 9 , 20 , 23 , 25（c）, 25（d）