Unit3. Inversion: Formed by DeMorgen's Law

DeMorgen's Law

$$(x+y)' = x'y' \qquad (xy)' = x'+y'$$

$$(x+y+z+\cdots)' = x'y'z'\cdots \qquad (xyz\cdots)' = x'+y'+z'+\cdots$$

$$\bullet \leftrightarrow + \quad , \quad A \leftrightarrow A'$$

Prove by truth table

Examples

1. $[(A'+B)C']' = (A'+B)'+C = AB'+C$

2. $[(AB'+C)D'+E']' = [(AB'+C)D']' \bullet E = [(AB'+C)'+D]E$
$= [(AB')'C'+D]E = [(A'+B)C'+D]E$
$= A'C'E + BC'E + DE$

General Form: (one-step rule)

$$[f(x_1, \; x_2, \; \cdots, \; x_n, \; 0, \; 1, \; +, \; \bullet)]' = f(x_1', \; x_2', \; \cdots, \; x_n', \; 1, \; 0, \; \bullet, \; +)$$

Example:

$$[(a'b+c')(d'+ef')+gh+w]' = [(a+b')c+d(e'+f)](g'+h')w'$$

Duality of a Boolean function

$AND \leftrightarrow OR \qquad 0 \leftrightarrow 1$

General Form:

$$[f(x_1,\ x_2,\ \cdots,\ x_n,\ 0,\ 1,\ +,\ \bullet)]^D = f(x_1,\ x_2,\ \cdots,\ x_n,\ 1,\ 0,\ \bullet,\ +)$$

$$Ex. \quad F = ab'+c+0\cdot d'(1+e)$$
$$F^D = (a+b')c(1+d'+0\cdot e)$$

1. *Firstly, form* F' 2. $a \leftrightarrow a'$

$$F' = (a'+b)c'\cdot(1+d+0\cdot e')$$
$$F^D = (a+b')c(1+d'+0\cdot e)$$

Property: $F = G \ \Rightarrow \ F^D = G^D$

$$Ex. \quad (x+y')y = xy \ \xrightarrow{Duality} \ x\cdot y'+y = x+y$$

Multiplying Out and Factoring Expressions

$$x(y+z) = xy + xz \quad \cdots(a)$$

$$x + yz = (x+y)(x+z) \quad \cdots(b)$$

Duality

$$(x+y)(x'+z) = xz + yx' \cdots (c)$$

$Pf \quad for \quad (c) \quad x=0 \quad$ 左 $= y \quad$ 右 $= y$

$x=1 \quad$ 左 $= z \quad$ 右 $= z$

Examples

1. $(Q+AB')(C'D+Q') = QC'D + Q'AB'$
2. $(A+B+C')(A+B+D)(A+B+E)(A+D'+E)(A'+C)$

$= [(A+B)+C'DE](AC+A'D'+A'E)$

$= AC + ABC + A'BD' + A'BE + A'C'DE$

Multiplying Out

Example cont'd.

3. $$AC + A'BD' + A'BE + A'C'DE$$

$$= AC + A'(BD' + BE + C'DE)$$

$$= (A + BD' + BE + C'DE)(A' + C)$$

$$= [\underbrace{A + C'DE}_{x} + \underbrace{B}_{y}(\underbrace{D' + E}_{z})](A' + C)$$

$$= (A + C'DE + B)(A + C'DE + D' + E)(A' + C)$$

$$= (A + B + C')(A + B + D)(A + B + E)(A + D' + E)(A' + C)$$

Exclusive-OR & Equivalence Operations

$\oplus$: *Exclusive–OR* $\qquad$ $+$: *Inclusive–OR*

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

$0 \oplus 0 = 0 \quad 0 \oplus 1 = 1$

$1 \oplus 0 = 1 \quad 1 \oplus 1 = 0$

$X \oplus Y = 1$ *if* $X = 1$ *or* $Y = 1$ *Not Both*

$X \oplus Y = XY' + X'Y$

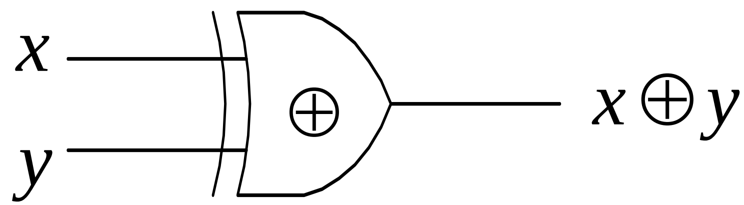
Properties:

$X \oplus 0 = X, \quad X \oplus 1 = X', \quad X \oplus Y = Y \oplus X$

$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$

$X(Y \oplus Z) = XY \oplus XZ$

$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$

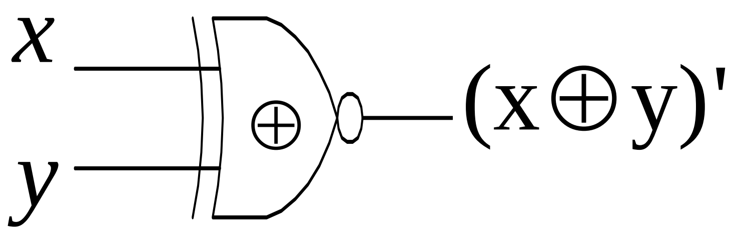


$\equiv$: Equivalence

X	Y	$X \equiv Y$
0	0	1
0	1	0
1	0	0
1	1	1

$$(0 \equiv 0) = 1 \quad (0 \equiv 1) = 0$$
$$(1 \equiv 0) = 0 \quad (1 \equiv 1) = 1$$

$$X \equiv Y = 1 \ \ if \quad X = Y = 1 \ \ or \quad X = Y = 0$$
$$\therefore (X \equiv Y) = XY + X'Y'$$



$$(X \oplus Y)' = (XY' + X'Y)' = (X + Y')(X' + Y) = XY + X'Y' = (X \equiv Y)$$

通常不希望於一 *Boolean Expression* 中有 ⊕，≡，以

$X \oplus Y = XY' + X'Y \qquad (X \equiv Y) = XY + X'Y'$ 代之

$$F = (A'B \equiv C) + (B \oplus AC')$$

$$= \left[(A'B)C + (A'B)'C'\right] + \left[B(AC')' + B'AC'\right]$$

$$= A'BC + AC' + B'C' + BA' + BC + AB'C'$$

$$= AC' + B'C' + BA' + BC = C'(A + B') + (A' + C)$$

又常用 $(xy' + x'y)' = xy + x'y' \quad or \quad xy' + x'y = (xy + x'y')'$

$$
\begin{aligned}
Ex.\quad A' \oplus B \oplus C &= (A'B' + AB) \oplus C \\
&= (A'B' + AB)C' + (A'B' + AB)'C \\
&= A'B'C' + ABC' + (AB' + A'B)C \\
&= A'B'C' + ABC' + AB'C + A'BC
\end{aligned}
$$

Positive & Negative Logic

$$\begin{array}{llll} Positive\quad Logic & 5V(H) & = & 1 \\ & 0V(L) & = & 0 \\ Negative\quad Logic & 5V(H) & = & 0 \\ & 0V(L) & = & 1 \end{array}$$

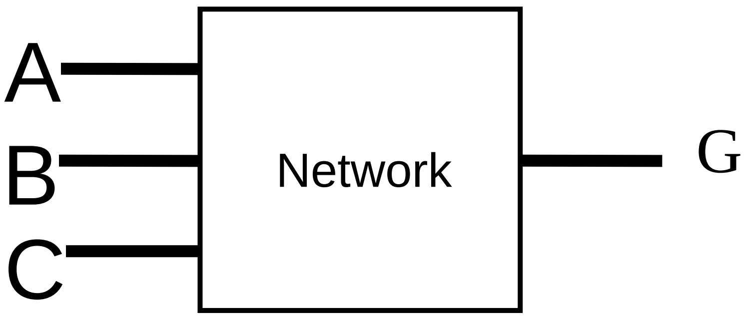
$$Positive\quad Logic \xleftrightarrow{Duality} Negative\quad Logic$$

+: Positive Logic -:Negative Logic

X (+)	X (level)	X (−)	Y (+)	Y (level)	Y (−)	AND XY (+, level)	XY (+)	OR (−)	OR $X+Y$ (+, level)	$X+Y$ (+)	AND (−)
0	L	1	0	L	1	L	0	1	L	0	1
0	L	1	1	H	0	L	0	1	H	1	0
1	H	0	0	L	1	L	0	1	H	1	0
1	H	0	1	H	0	H	1	0	H	1	0

$$\Rightarrow \quad (+)AND = (-)OR$$
$$(+)1 = (-)0$$
$$(+)OR = (-)AND$$

Ex. Positive Logic $\xleftrightarrow[\text{Duality}]{}$ Negative Logic



$$if \quad Network \quad is \quad +Logic$$
$$G = ABC' + A'B'C$$
$$if \quad Network \quad is \quad -Logic$$
$$G = (A+B+C')(A'+B'+C)$$

Concensus Theorem : $xy + x'z + yz = xy + x'z$

pf : $xy + x'z + yz$

$= xy + x'z + (x + x')yz$

$= (xy + xyz) + (x'z + xyz)$

$= xy(1 + z) + x'z(1 + y)$

$= xy + x'z$

Dual form : $(x + y)(x' + z)(y + z) = (x + y)(x' + z)$

Ex : 1. $a'b' + ac + bc' + b'c + ab = (a'b' + ac + bc')$

2. $(a + b + c')(a + b + d')(b + c + d')$

$= (a + b + c')(b + c + d')$

Algebraic Simplification
Multiplying
Factoring

1. Combining terms

$$use \quad xy + xy' = x$$

Examples

1. $abc'd' + abcd' = abd'$
2. $ab'c + abc + a'bc = ab'c + abc + abc + a'bc = ac + bc$
3. $\underbrace{(a + bc)}_{Y}\underbrace{(d + e')}_{X} + \underbrace{a'(b' + c')}_{Y'}\underbrace{(d + e')}_{X} = d + e'$

2. Eliminating terms

$$Use \quad (a) \quad x + xy = x$$

$$(b) \quad Concensus \;\; Th's$$

$$xy + x'z + yz = xy + x'z$$

Examples

1. $a'b + a'bc = a'b$

2. $a'bc' + bcd + a'bd = a'bc' + bcd$

3. Eliminating literals

Use $x + x'y = x + y$

Examples

$$A'B + A'B'C'D' + ABCD' = A'(B + B'C'D') + ABCD' = A'(B + C'D') + ABCD'$$
$$= B(A' + ACD') + A'C'D' = B(A' + CD') + A'C'D' = A'B + BCD' + A'C'D'$$

4. Adding redundant terms

Add xx', yz to $xy + x'z$, xy to x, Multiply $(x + x')$

Examples

$$wx + xy + x'z' + wy'z' = wx + xy + x'z' + wy'z' + wz'$$
$$= wx + xy + x'z' + wz' = wx + xy + x'z'$$

Use all 4 methods

Example: ① SOP

$$\underbrace{A'B'C'D' + A'BC'D'}_{❶} + \underbrace{A'BD + A'BC'D}_{❷} + ABCD + ACD' + B'CD'$$

$$= A'C'D' + BD\underbrace{(\forall' + \forall' C)}_{❸\ A'+C} + ACD' + B'CD'$$

$$= A'C'D' + A'BD + \underbrace{BCD + ACD'}_{+ABC\ ❹} + B'CD'$$

$$= A'C'D' + \underbrace{\forall' BD + B\not{C}D + A\not{C}D' + \overbrace{B'CD' + \forall' BC}^{Concensus\ \ ACD'}}_{Concensus\ \ BCD}$$

$$= A'C'D' + A'BD + B'CD' + ABC$$

Example:② POS

$$\underbrace{(A'+B'+C')(A'+B'+C)}_{❶}(B'+C)(A+C)\underbrace{(A+B+C)}_{❷}$$

Use duals of the theorem

$$=(A'+B')\underbrace{(B'+C)}_{❸}(A+C)$$

$$=(A'+B')(A+C)$$

↑ *Concensus Th.*

證明恆等式

Example 1.

prove $A'BD' + BCD + ABC' + AB'D$

$= BC'D' + AD + A'BC$ is valid

方法:

1. 建造Truth Table,for 左邊 &右邊

2.化簡左右邊使之相等

$$A'BD' + BCD + ABC' + AB'D + BC'D' + A'BC + ABD$$

$$A'BD' \quad A'BD' \quad BCD$$

$$ABC' \quad BCD \quad ABC'$$

$$AD + A'BD' + BCD + ABC' + BC'D' + A'BC$$

$$= BC'D' + AD + A'BC$$

Example 2.

prove $A'BC'D + (A'+BC)(A+C'D') + BC'D + A'BC'$

$$= ABCD + A'C'D' + ABD + ABCD' + BC'D$$

左邊 $= (A'+BC)(A+C'D') + A'BC'D + BC'D + A'BC'$

$= (A'+BC)(A+C'D') + BC'D + A'BC'$

$= ABC + A'C'D' + BC'D + A'BC'$

$= ABC + A'C'D' + BC'D$

右邊 $= ABC + A'C'D' + \underbrace{ABD}_{\text{concensus}} + BC'D$

$= ABC + A'C'D' + BC\ D$

注意: In Boolean algebra

True for ordinary algebra, but not true for Boolean algebra

$$x+y=x+z \not\Rightarrow y=z$$

$$\because\ x=1,\ y=0,\ z=1 \Rightarrow 1+0=1+1 \Rightarrow 0=1$$

$$xy = xz \not\Rightarrow y = z$$

$$\because\ x = 0,\ y = 1,\ z = 0,$$

$$\Rightarrow 0.01 = 0.0 \Rightarrow 0 = 1$$

Homework

3.6	3.11
3.7	3.19
3.8	3.22 (a) (d) (g)
3.10	3.25 (b) (e)
	3.28 (a)

Supplement for p. 17 examples

A’B’C’D’+A’BC’D’+A’BD+A’BC’D+ABCD+ACD’+B’CD’

--------- --------- (get common term A’C’D’, A’C’D’(B’+B))

------- -------- ------- (Get common term BD, BD (A’+A’C’+AC))

\- ---- = A’

= A’C’D’+BD(A’+AC)+ACD’+B’CD’

------------ (expand)

= A’C’D’+A’BD+ABCD+ACD’+B’CD’

= A’C’D’+A’BD+(ABCD+ACD’+ABC)+B’CD’

------- ----- ^^^^ (Consensus theorem, take D, then combine ABCD+ABC)

= A’C’D’+A’BD+(ACD’+ABC+B’CD’)

^^^^ ------ ------ (Consensus ACD’, take B,)

= A’C’D’+A’BD+ABC+B’CD’