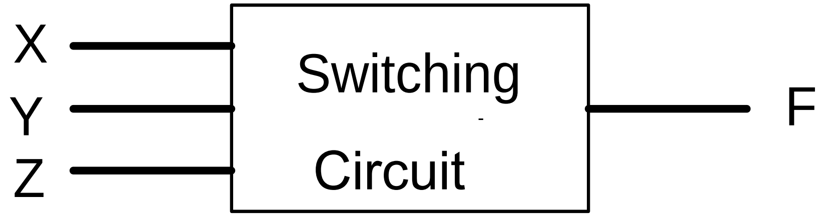
Unit 2 Boolean Algebra

Basic Operations

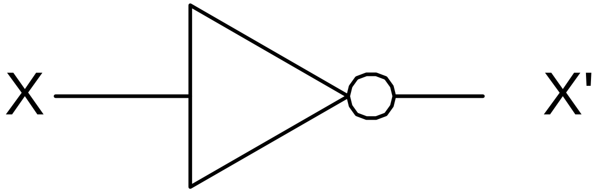
Boolean Variables: X, Y, Z … takes "0", "1"

"L", "H"

"F", "T"

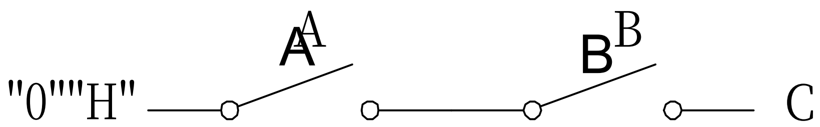


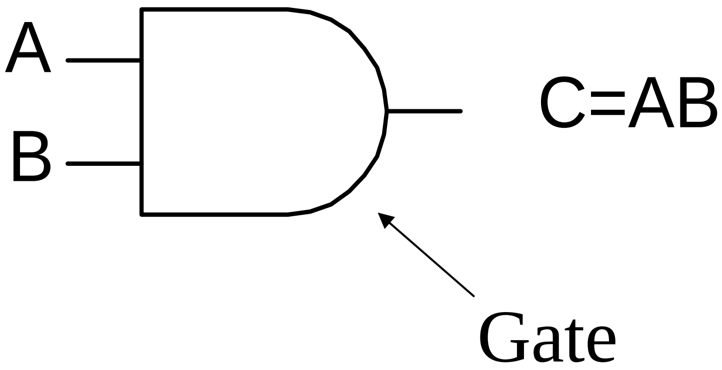
$$Complement: \quad 0 \leftrightarrow 1 \quad x \leftrightarrow x' \quad (or \quad x \leftrightarrow \overline{x})$$



$$AND \qquad 0\cdot 0 = 0 \quad 0\cdot 1 = 0 \quad 1\cdot 0 = 0 \quad 1\cdot 1 = 1$$

Switch

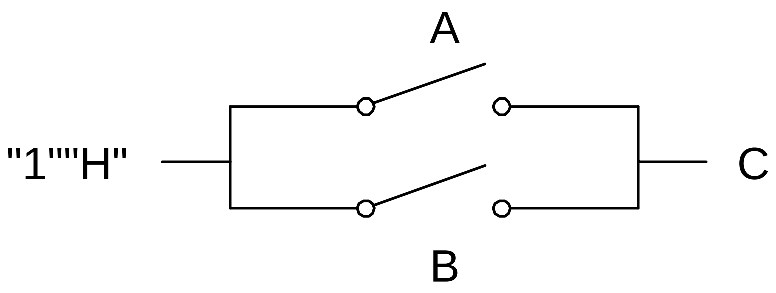


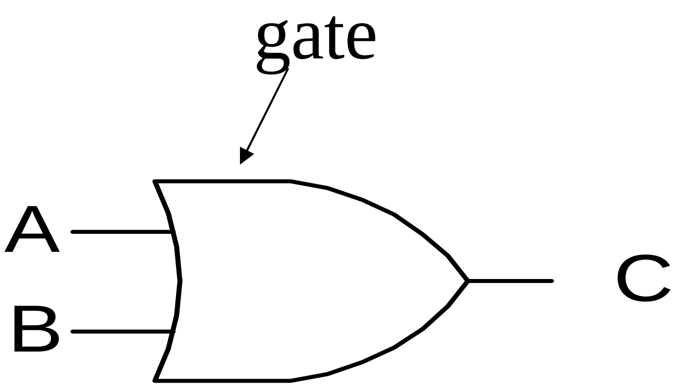


A	B	$C = A\cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table

$$OR \quad 0+0=0 \quad 0+1=1 \quad 1+0=1 \quad 1+1=1$$



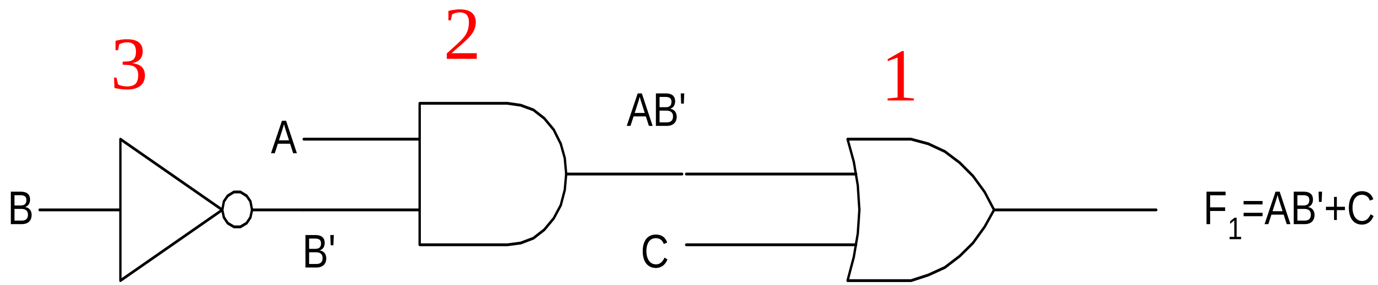


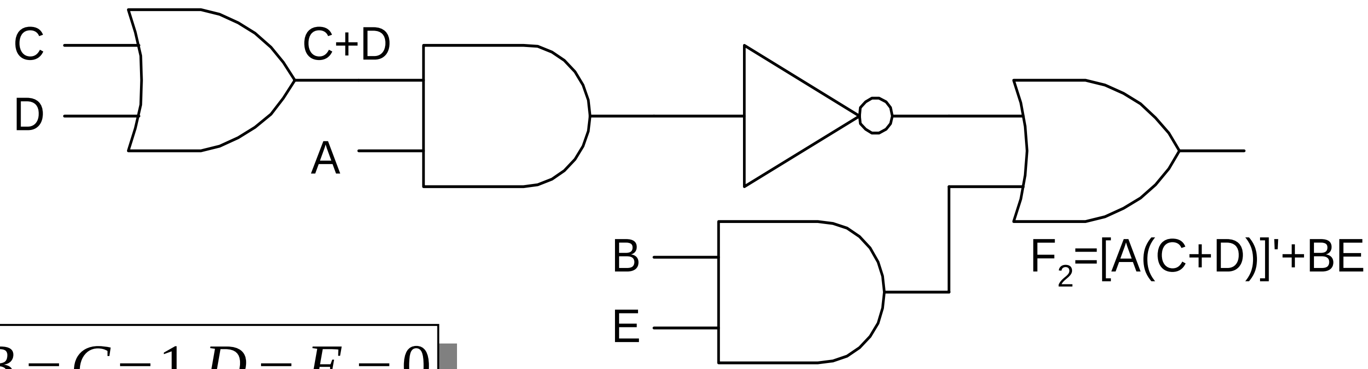
A	B	$C=A+B$
0	0	0
0	1	1
1	0	1
1	1	1

Truth Table

Boolean Expressions and Truth Tables

$$Boolean\ \ Expression:\quad F_1 = AB' + C \quad F_2 = [A(C+D)]' + BE$$





$$if \quad A = B = C = 1, D = E = 0$$

$$\Rightarrow \quad F_1 = 1 \cdot 0 + 1 = 1$$

$$F_2 = [1(1+0)]' + 1 \cdot 0 = 0$$

Truth Table

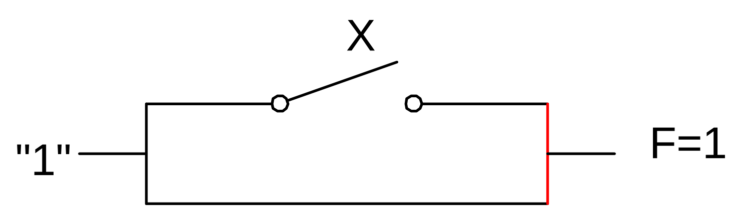
$$2^3 \quad \left(2^n\right)$$

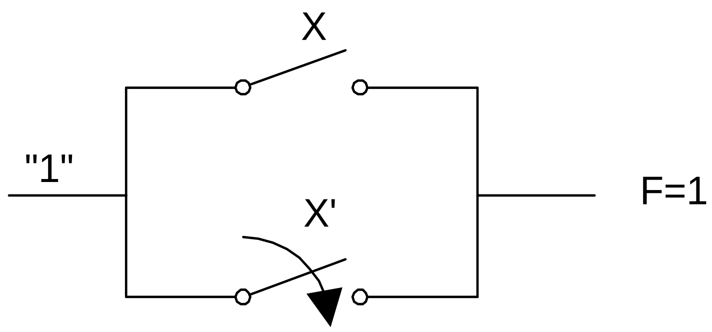
A	B	C	B'	AB'	$AB'+C$	$A+C$	$B'+C$	$(A+C)(B'+C)$
0	0	0	1	0	0	0	1	0
0	0	1	1	0	1	1	1	1
0	1	0	0	0	0	0	0	0
0	1	1	0	0	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	0	1	0	0
1	1	1	0	0	1	1	1	1

Basic Theorems and Laws of Boolean Algebra

$x+0$	$=$	x
$x+1$	$=$	1
$x+x$	$=$	x
$(x')'$	$=$	x
$x+x'$	$=$	1

$x\cdot 1$	$=$	x
$x\cdot 0$	$=$	0
$x\cdot x$	$=$	x
$x\cdot x'$	$=$	0





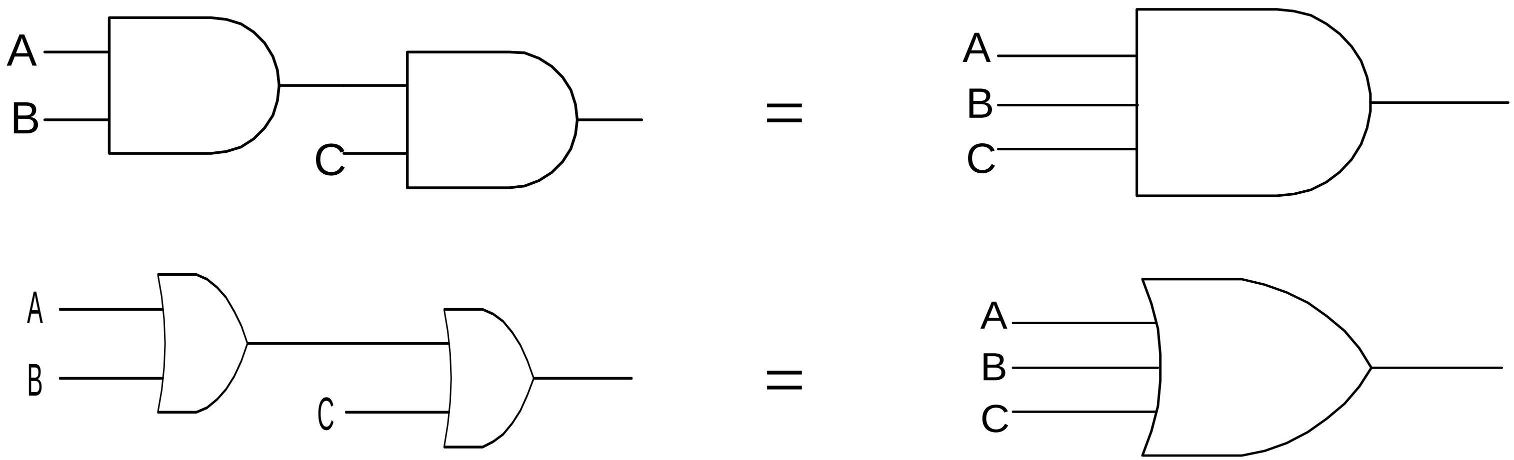
$$\Rightarrow \quad (AB'+D)E+1 = 1$$

$$(AB'+D)(AB'+D)' = 0$$

Prove?

$$\textit{Commutative Law} \qquad xy = yx \qquad x + y = y + x$$

$$\textit{Associative Law} \quad \begin{cases} (xy)z = x(yz) = xyz \\ (x+y)+z = x+(y+z) = x+y+z \end{cases}$$



$$\Rightarrow \quad xyz = 1 \quad \Rightarrow \quad x = 1 = y = z$$

$$x + y + z = 0 \quad \Rightarrow \quad x = 0 = y = z$$

$$x(y+z) = xy + xz$$

AND distributes over OR

$$x + yz = (x+y)\cdot(y+z)$$

Distributive Law

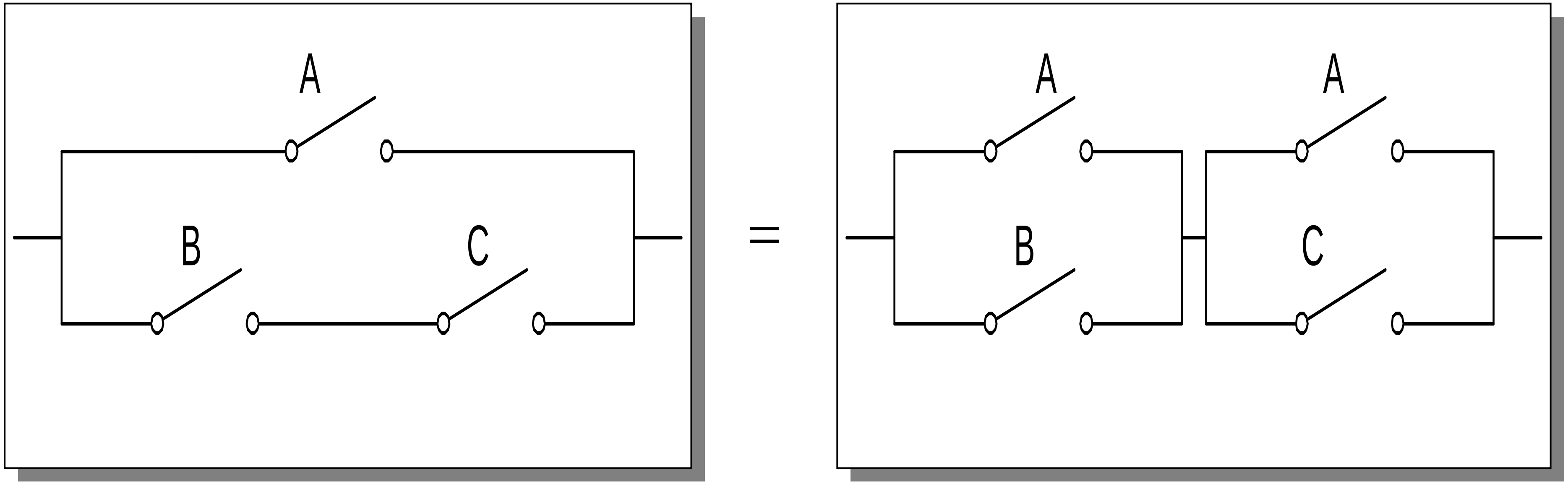
OR distributes over AND

Only valid for Boolean Algebra

$pf:$ 右邊 $(x+y)\cdot(x+z) = x\cdot(x+z) + y(x+z) = x\cdot x + xz + yx + yz$

$= x + xz + yx + yz = x(1+z+y) + yz$

$= x + yz$



Simplification Theorems

$\leftarrow$ Duality $\rightarrow$

$$xy + xy' = x \qquad\qquad (x+y)(x+y') = x \quad \cdots 2$$

$$x + xy = x \quad \cdots 1 \qquad\qquad x(x+y) = x$$

$$(x+y')y = xy \qquad\qquad xy' + y = x + y$$

pf 1. $x + xy = x(1+y)$
$= x$

2. $(x+y)(x+y') = x + yy'$
$= x + 0$

$$
\begin{aligned}
Ex.\quad 1.\quad Z &= A'BC + A' \\
&= A'(1+BC) \\
&= A'
\end{aligned}
$$

$$
\begin{aligned}
Ex.\quad 2\quad Z &= \left[\underbrace{A+B'C}_{x} \quad + \quad \underbrace{D+EF}_{y}\right]\left[\underbrace{A+B'C}_{x} \quad + \quad \underbrace{(D+EF)'}_{y'}\right] \\
&= \underbrace{A+B'C}_{x}
\end{aligned}
$$

$$
\begin{aligned}
Ex.\quad 3.\quad Z &= \left(\underbrace{AB+C}_{y'}\right)\left(\underbrace{B'D+C'E'}_{x}\right)+\left(\underbrace{(AB+C)'}_{y}\right) \\
&= x+y \\
&= B'D+C'E'+(AB+C)'
\end{aligned}
$$

Multiplying out and Factoring

Sum of Product Form :

$$ABC + A'B'C + ABC'$$
$$AB' + CD'E + AC'E'$$
$$ABC' + DEFG + H$$
$$A + B' + C + D'E$$

(A+B)CD+EF not SOP form

Why do you need SOP or POS form ?

And How?

Multiplying Out: Get expressions to be SOP form

When multiplying out:

Use (A+B)(A+C) = A + BC

Ex.

$$\left(A + \underbrace{BC}_{x}\right)\left(A + \underbrace{D+E}_{y}\right) = A + xy$$

$$= A + BC(D+E)$$

$$= A + BCD + BCE$$

Product of Sum form (POS)

$$(A+B')(C+D'+E)(A+C'+E')$$

$$(A+B)(C+D+E)F$$

$$AB'C(D'+E)$$

Factoring: get expression to be POS form

Use: $x+yz = (x+y)(x+z)$

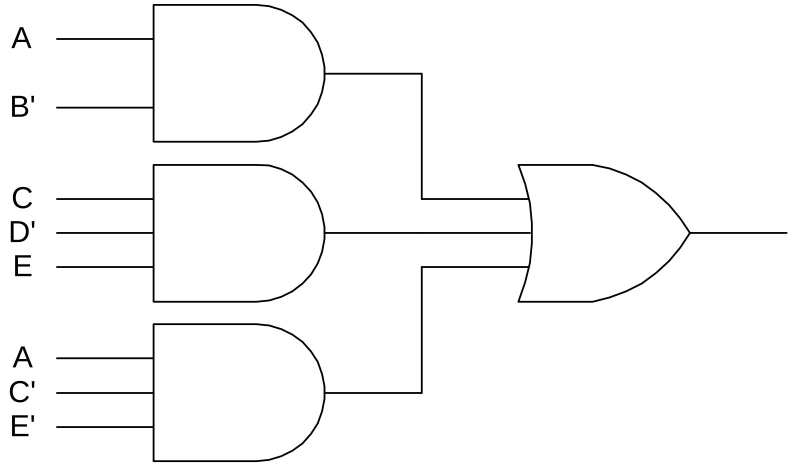
Product of Sum form (POS): Example

$$
\begin{aligned}
Ex \quad 1. \quad & A + B'CD &&= (A+B')(A+C)(A+D) \\
2. \quad & AB' + C'D &&= (AB'+C')(AB'+D) \\
& &&= (A'+C')(B'+C')(A+D)(B'+D) \\
3. \quad & C'D + C'E' + G'H &&= C'(D+E') + G'H \\
& &&= (C'+G'H)(D+E'+G'H) \\
& &&= (C'+G')(C'+H)(D+E'+G')(D+E'+H)
\end{aligned}
$$

Realization: 2-level realization

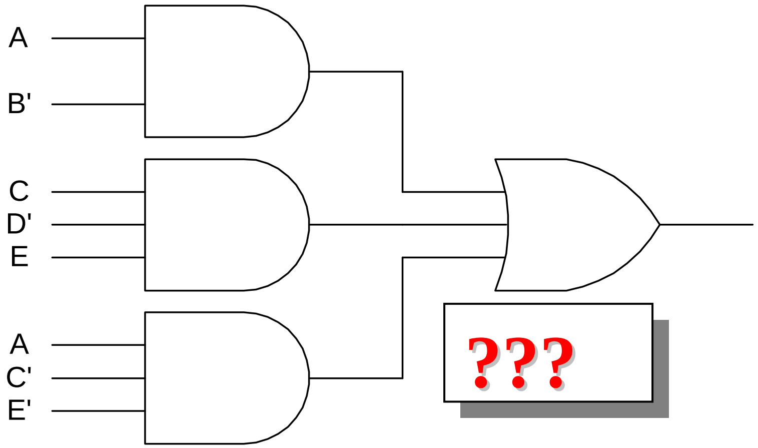
SOP

$$AB'+CD'E+AC'E'$$



POS

$$(A+B')(C+D'+E)(A+C'+E')$$



???

A', B', C' Use Inverter

DeMorgan's Laws

$(X+Y)'=X'Y'$

$(XY)'=X'+Y'$

Homework

2.2 2.5 2.7(b)(c) 2.9 2.13 2.15 2.17 2.20 2.22 2.28