Chap. 15:

Reduction of State Tables/ State Assignment

Statement of Problen

⇓

State Graph

⇓

State Table

⇓

Reduction of States

⇓

State Assignment

⇓

Choice of F/F's

⇓

Derivation of
F/F Input Equation and
Z Output Equation

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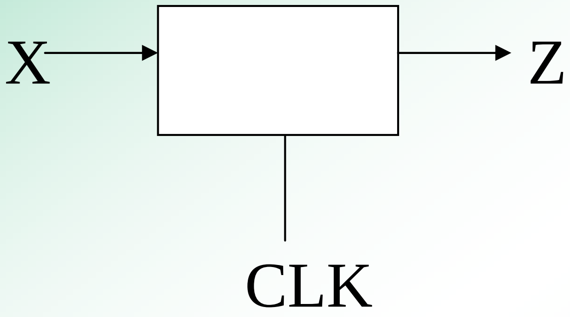
Circuit Realization and
Timing Chart

15-1 Elimination of Redundant States

When setting up states, some extra states may be included.

$\Rightarrow$ eliminate these states

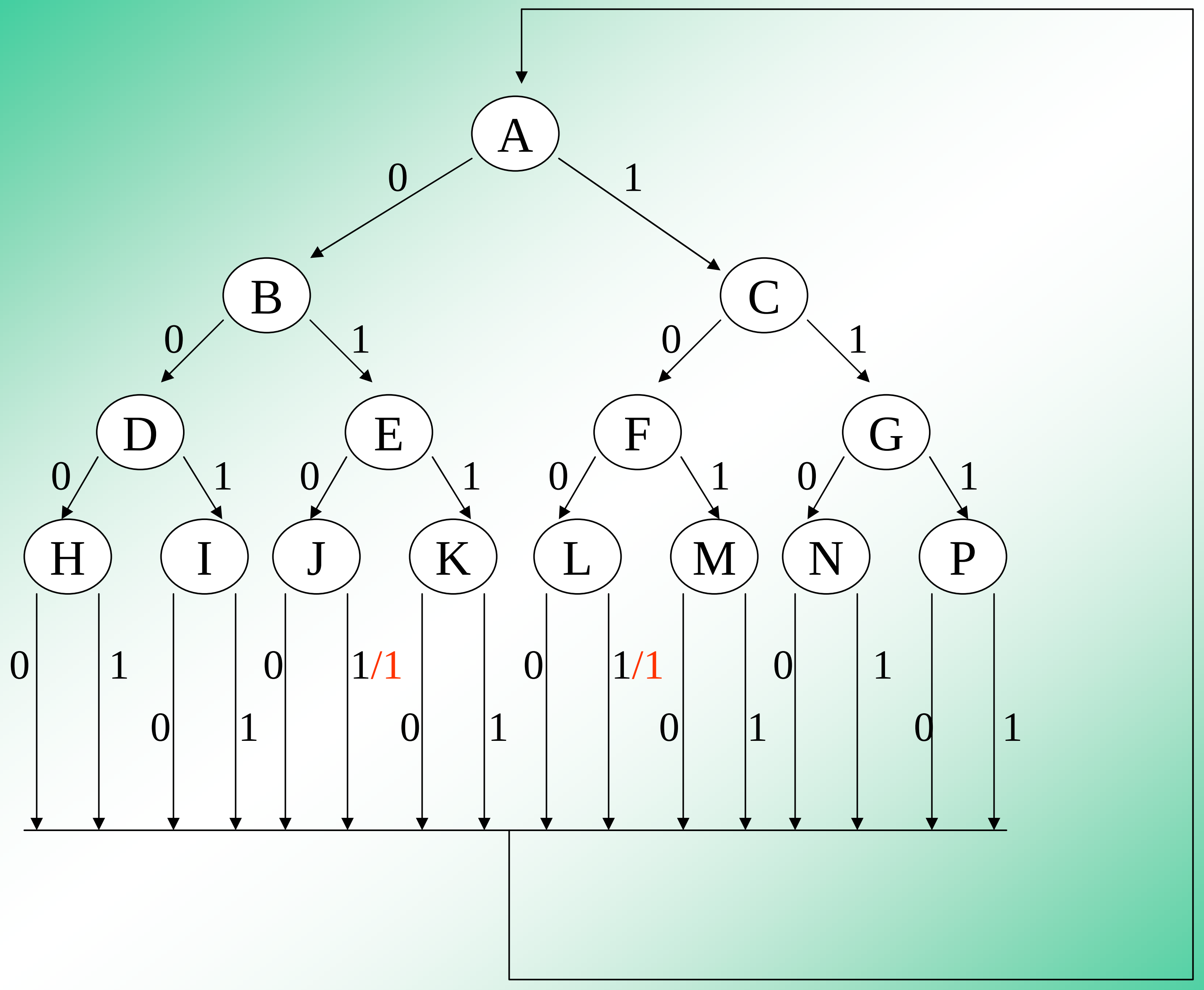
Ex:a previous example



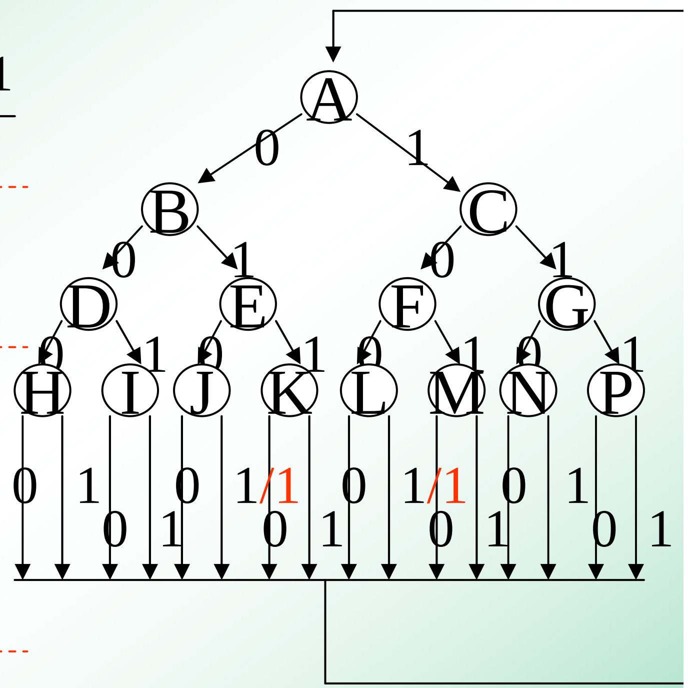
Z gives "1" after X receiving 0101 or 1001

A brute force solution:

Reset state A, checks 3 consequent bits of every possible combinations.
After 4th bit coming-in, give output and reset to state A.


A
B
C
D
E
F
G
H
I
J
K
L
M
N
P
0
1
1/1

Input Sequence	P.S.	N.S. x = 0	N.S. x = 1	Present Output x = 0	Present Output x = 1
Reset	A	B	C	0	0
0	B	D	E	0	0
1	C	F	G	0	0
00	D	H	I	0	0
01	E	J	K	0	0
10	F	L	M	0	0
11	G	N	P	0	0
000	H	A	A	0	0
001	I	A	A	0	0
010	J	A	A	0	1
011	K	A	A	0	0
100	L	A	A	0	1
101	M	A	A	0	0
110	N	A	A	0	0
111	P	A	A	0	0

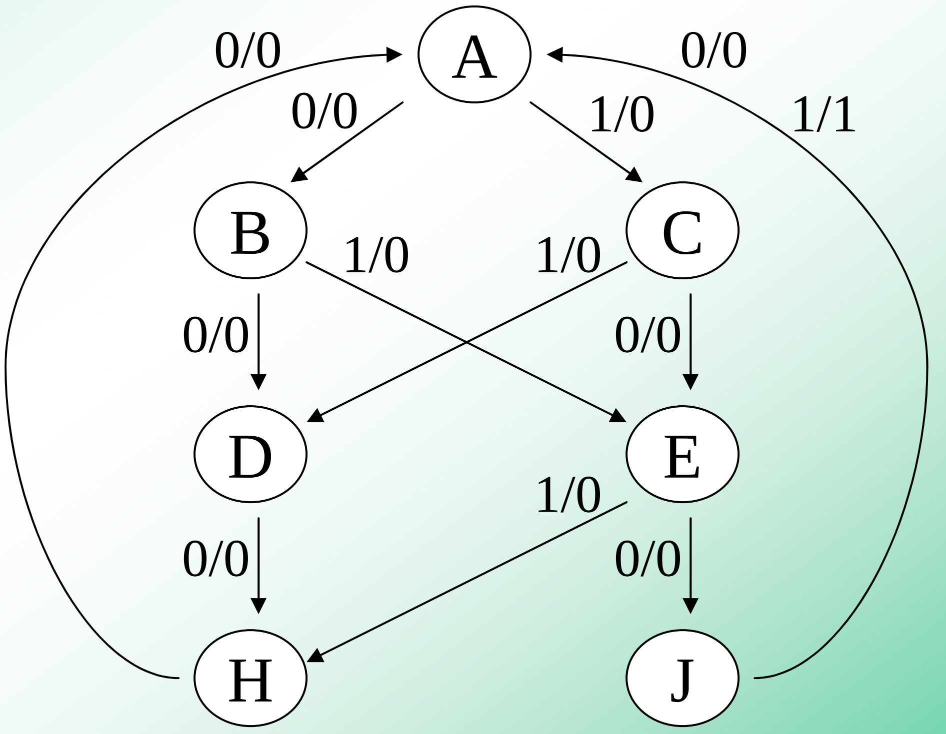


Input Sequence	P.S.	N.S. x = 0	N.S. x = 1	Present Output x = 0	Present Output x = 1
Reset	A	B	C	0	0
0	B	D	E	0	0
1	C	E ~~F~~	D ~~G~~	0	0
00	D	H	H ~~I~~	0	0
01	E	J	H ~~K~~	0	0
→ 10	E ~~F~~	J ~~L~~	H ~~M~~	0	0
→ 11	D ~~G~~	H ~~N~~	H ~~R~~	0	0
000	H	A	A	0	0
→ 001	H ~~I~~	A	A	0	0
010	J	A	A	0	1
→ 011	H ~~K~~	A	A	0	0
→ 100	J ~~L~~	A	A	0	1
→ 101	H ~~M~~	A	A	0	0
→ 110	H ~~N~~	A	A	0	0
→ 111	H ~~R~~	A	A	0	0

Equivalent state :

- ⊙ The same N.S.
- ⊙ The same Z

P.S.	N.S. x = 0	N.S. x = 1	P.O. x = 0	P.O. x = 1
A	B	C	0	0
B	D	E	0	0
C	E	D	0	0
D	H	H	0	0
E	J	H	0	0
H	A	A	0	0
J	A	A	0	1

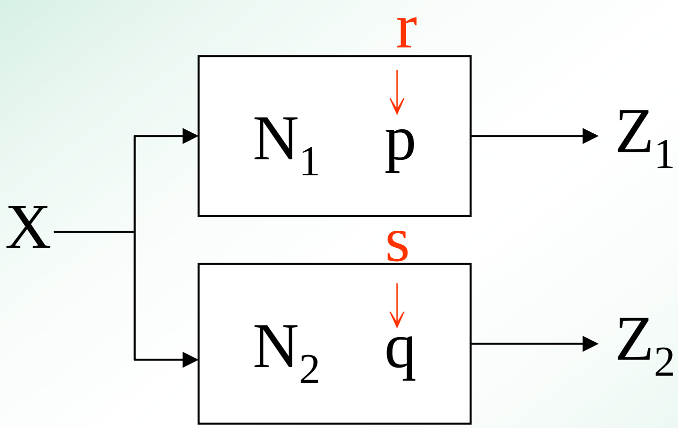


15-2 Equivalent States

Equivalent States " ≡ "

2 machines : N_1, N_2

$$\begin{cases} \text{A state } p \text{ in } N_1 \\ \text{A state } q \text{ in } N_2 \end{cases}$$



If $Z_1 = Z_2$ and $r \equiv s$

then $p \equiv q$

$\lambda(p, x) = \lambda(q, x)$: output

$\delta(p, x) = \delta(q, x)$: next state

15-3 Determination of State Equivalence by using an Implication Table

Ex:

P.S.	N.S. x = 0	N.S. x = 1	Z
a	~~d~~ a	c	0
b	f	h	0
c	~~e~~ c	~~d~~ a	1
~~d~~	~~a~~	~~e~~	~~0~~
~~e~~	~~c~~	~~a~~	~~1~~
f	f	b	1
g	b	h	0
h	c	g	1

Implication chart construction

1. state comparison by implied pair (only for the same output)

P.S.	N.S. x = 0	N.S. x = 1	Z
a	d	c	0
b	f	h	0
c	e	d	1
d	a	e	0
e	c	a	1
f	f	b	1
g	b	h	0
h	c	g	1

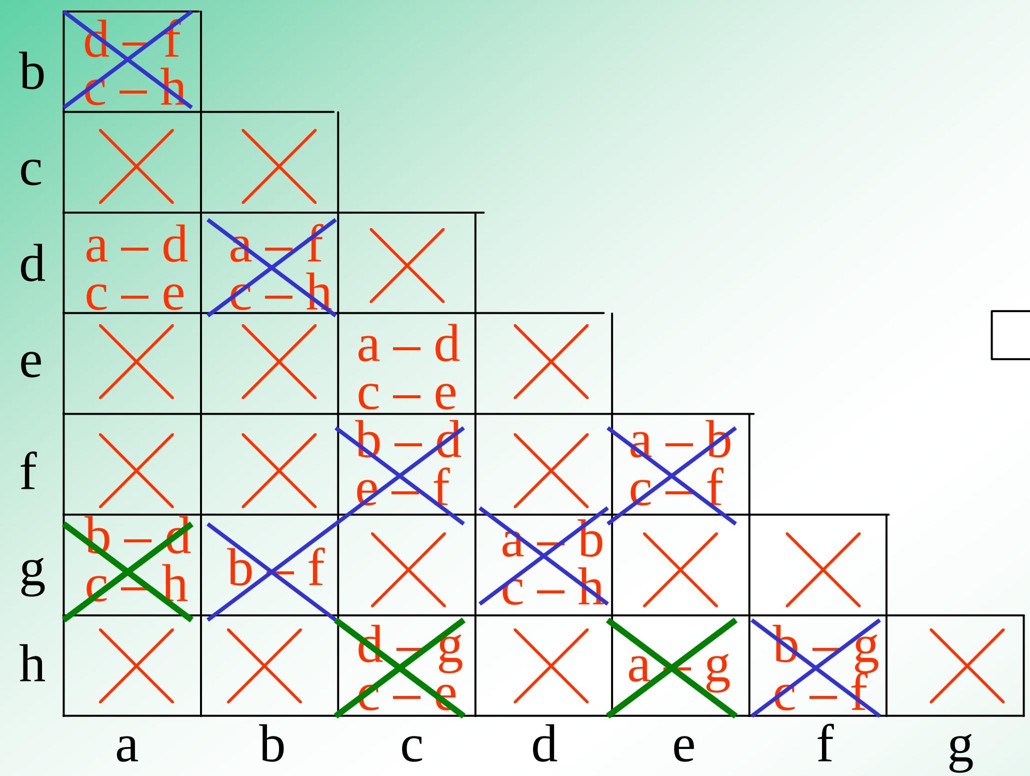
Output shall be the same

	a	b	c	d	e	f	g
b	d – f c – h						
c	X	X					
d	a – d c – e	a – f c – h	X				
e	X	X	a – d c – e	X			
f	X	X	b – d e – f	X	a – b c – f		
g	b – d c – h	b – f	X	a – b c – h	X	X	
h	X	X	d – g c – e	X	a – g	b – g c – f	X

2. Check implied pair iteratively

	a	b	c	d	e	f	g
b	~~d – f c – h~~						
c	X	X					
d	a – d c – e	~~a – f c – h~~	X				
e	X	X	a – d c – e	X			
f	X	X	~~b – d e – f~~	X	~~a – b c – f~~		
g	~~b – d c – h~~	~~b – f~~	X	~~a – b c – h~~	X	X	
h	X	X	~~d – g c – e~~	X	~~a – g~~	~~b – g c – f~~	X

P.S.	N.S. x = 0	N.S. x = 1	Z
a	~~d~~ a	c	0
b	f	h	0
c	~~e~~ c	~~d~~ a	1
~~d~~	~~a~~	~~e~~	~~0~~
~~e~~	~~c~~	~~a~~	~~1~~
f	f	b	1
g	b	h	0
h	c	g	1



$a \equiv d$

$c \equiv e$

P.S.	N.S. x = 0	N.S. x = 1	Z
a	a	c	0
b	f	h	0
c	c	a	1
f	f	b	1
g	b	h	0
h	c	g	1

15-4 Equivalent Sequential Circuits

Definition : $N_1 \equiv N_2$ iff P in $N_1 \equiv$ q in N_2
S in $N_2 \equiv$ t in N_1

Ex:

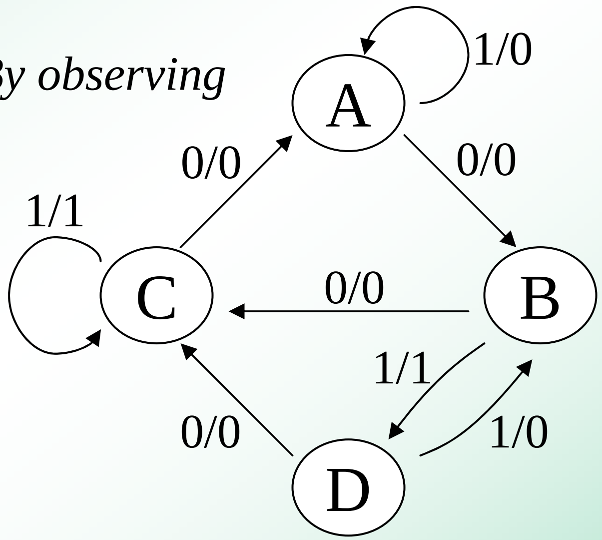
N_1

	x = 0	x = 1	x = 0	x = 1
A	B	A	0	0
B	C	D	0	1
C	A	C	0	1
D	C	B	0	0

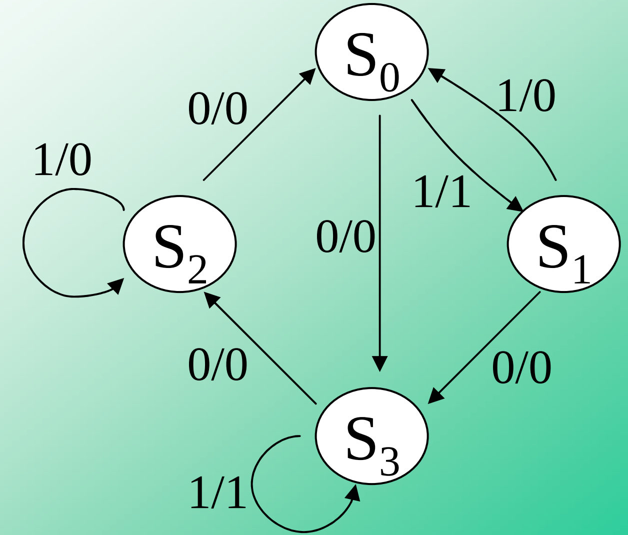
N_2

	x = 0	x = 1	x = 0	x = 1
S_0	S_3	S_1	0	1
S_1	S_3	S_0	0	0
S_2	S_0	S_2	0	0
S_3	S_2	S_3	0	1

Method 1. By observing



$A \equiv S_2$
$B \equiv S_0$
$C \equiv S_3$
$D \equiv S_1$



N_1

	$x=0$	$x=1$	$x=0$	$x=1$
A	B	A	0	0
B	C	D	0	1
C	A	C	0	1
D	C	B	0	0

N_2

	$x=0$	$x=1$	$x=0$	$x=1$
S_0	S_3	S_1	0	1
S_1	S_3	S_0	0	0
S_2	S_0	S_2	0	0
S_3	S_2	S_3	0	1

Method 2. By implication table

1. Construct implication table by listing all state pair. X out pairs with different output

	A	B	C	D
S_0	×	$C-S_3$ $D-S_1$	$A-S_3$ $C-S_1$	×
S_1	$B-S_3$ $A-S_0$	×	×	$C-S_3$ $B-S_0$
S_2	$B-S_0$ $A-S_2$	×	×	$C-S_0$ $B-S_2$
S_3	×	$C-S_2$ $D-S_3$	$A-S_2$ $C-S_3$	×

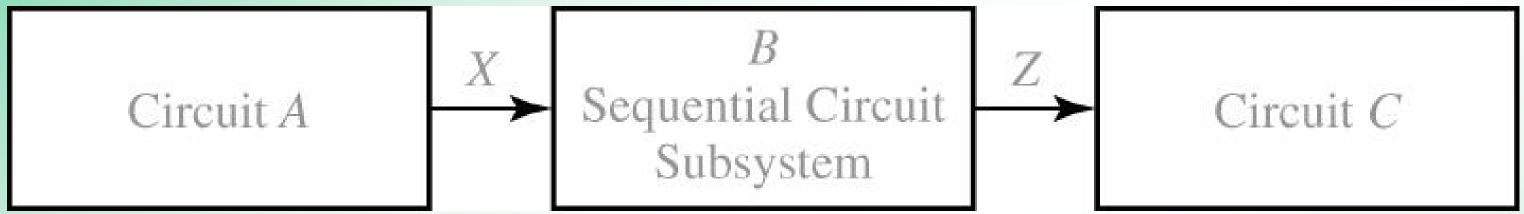
	A	B	C	D
S_0	X	$C - S_3$ $D - S_1$	$A - S_3$ $C - S_1$	X
S_1	$B - S_3$ $A - S_0$	X	X	$C - S_3$ $B - S_0$
S_2	$B - S_0$ $A - S_2$	X	X	$C - S_0$ $B - S_2$
S_3	X	$C - S_2$ $D - S_3$	$A - S_2$ $C - S_3$	X

	A	B	C	D
S_0	X	$C - S_3$ $D - S_1$	~~$A - S_3$~~ ~~$C - S_1$~~	X
S_1	~~$B - S_3$~~ ~~$A - S_0$~~	X	X	$C - S_3$ $B - S_0$
S_2	$B - S_0$ $A - S_2$	X	X	~~$C - S_0$~~ ~~$B - S_2$~~
S_3	X	~~$C - S_2$~~ ~~$D - S_3$~~	$A - S_2$ $C - S_3$	X

$$\Rightarrow \quad A \equiv S_2 \quad B \equiv S_0 \quad C \equiv S_3 \quad D \equiv S_1$$

15-5 Incompletely Specified State Tables

Ex:



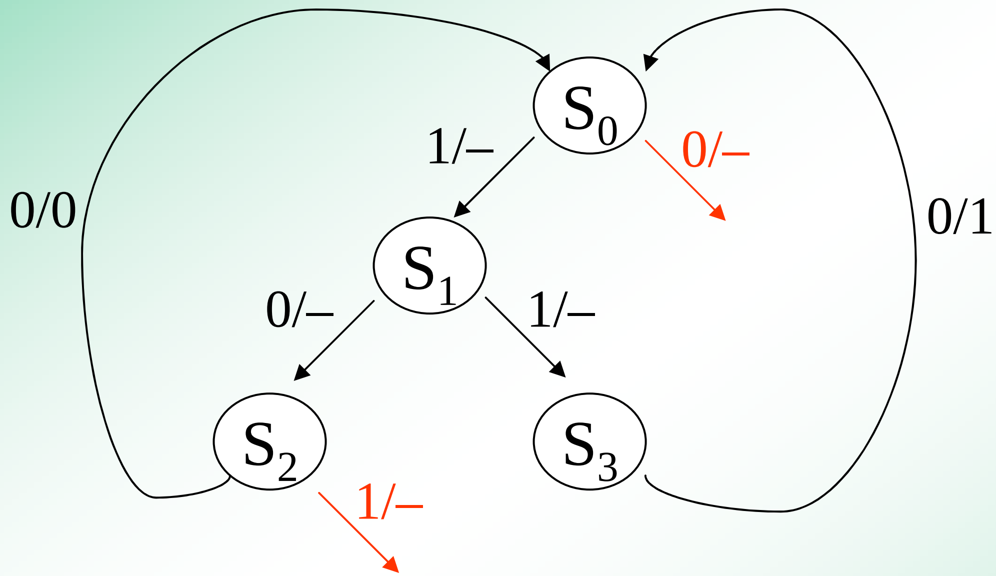
$$\text{A can only gives } 100\,,110$$

$$\text{B gives} \begin{Bmatrix} \text{"0"} & \text{if} & 100 \ \text{received} \\ \text{"1"} & & 110 \ \text{received} \end{Bmatrix} \text{at 3rd bit}$$

	t_0	t_1	t_2		t_0	t_1	t_2
X =	1	0	0	Z =	–	–	***0***
	1	1	0		–	–	***1***

If don't cares are present, the state table is incompletely specified

State Graph for Network B:



P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
S_0	–	S_1	–	–
S_1	S_2	S_3	–	–
S_2	S_0	–	***0***	–
S_3	S_0	–	***1***	–

P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
S_0	– (S_0)	S_1	– (0)	–
S_1	~~S_2~~ S_0	–	– (1)	–
→ S_2	S_0	– (S_1)	***0***	–
→ S_3	S_0	–	***1***	–

⇩

P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
S_0	S_0	S_1	***0***	–
S_1	S_0	–	***1***	–

Reduction of states !!

15-6 Derivation of F/F Input Equations

State Graph

$\Rightarrow$ State Table

$\Rightarrow$ State Assignment

$\Rightarrow$ State Transition Table

Choice of F/F!

$\Rightarrow$ K-map

$\Rightarrow$ F/F Input Equations

Ex :

P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
S_0	S_1	S_2	0	0
S_1	S_3	S_2	0	0
S_2	S_1	S_4	0	0
S_3	S_5	S_2	0	0
S_4	S_1	S_6	0	0
S_5	S_5	S_2	1	0
S_6	S_1	S_6	0	1

⇩

$$7 \text{ states} \Rightarrow 3 \text{ F/F's} \quad A\ B\ C$$

⇩

$$\text{Let } S_0 = 000, S_1 = 110, S_2 = 001, S_3 = 111,$$
$$S_4 = 011, S_5 = 101, S_6 = 010$$

⇩

Transition Table

A	B	C	$A^+B^+C^+$ $x=0$	$A^+B^+C^+$ $x=1$	Z $x=0$	Z $x=1$
0	0	0	110	001	0	0
1	1	0	111	001	0	0
0	0	1	110	011	0	0
1	1	1	101	001	0	0
0	1	1	110	010	0	0
1	0	1	101	001	1	0
0	1	0	110	010	0	1

(1) Choose D-F/F

Q^{+}	D
0	0
1	1

ABC	D_A X=0	D_A X=1	D_B X=0	D_B X=1	D_C X=0	D_C X=1
000	1	0	1	0	0	1
110	1	0	1	0	1	1
001	1	0	1	1	0	1
111	1	0	0	0	1	1
011	1	0	1	1	0	0
101	1	0	0	0	1	1
010	1	0	1	1	0	0

$\Rightarrow$

ABC	D_A X=0	D_A X=1	D_B X=0	D_B X=1	D_C X=0	D_C X=1
000	1	0	1	0	0	1
001	1	0	1	1	0	1
010	1	0	1	1	0	0
011	1	0	1	1	0	0
100	×	×	×	×	×	×
101	1	0	0	0	1	1
110	1	0	1	0	1	1
111	1	0	0	0	1	1

ABC	D_A X=0	D_A X=1	D_B X=0	D_B X=1	D_C X=0	D_C X=1
000	1	0	1	0	0	1
001	1	0	1	1	0	1
010	1	0	1	1	0	0
011	1	0	1	1	0	0
100	×	×	×	×	×	×
101	1	0	0	0	1	1
110	1	0	1	0	1	1
111	1	0	0	0	1	1

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

$$A^{+} = D_A = X'$$

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	0	0	1
11	1	0	0	1
10	1	1	0	1

$$B^{+} = D_B$$
$$= X'C' + A'C + A'B$$

BC \ XA	00	01	11	10
00	0	×	×	1
01	0	1	1	1
11	0	1	1	0
10	0	1	1	0

$$C^{+} = D_C$$
$$= A + XB'$$

Derivation of D F/F input equations

(2) Choose J-K F/F

Q	Q^+	J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

ABC	$A^+B^+C^+$ X=0	X=1	J_AK_A X=0	X=1	J_BK_B X=0	X=1	J_CK_C X=0	X=1
000	110	001	1×	0×	1×	0×	0×	1×
110	111	001	×0	×1	×0	×1	1×	1×
001	110	011	1×	1×	:	:	:	:
111	101	001	×0	×1	:	:	:	:
011	110	010	1×	×0	:	:	:	:
101	101	001	×0	0×	:	:	:	:
010	110	010	×0	×0	:	:	:	:

Derivation of Flip-Flop Input Equations from Next State Maps

		Q=0		Q=1		Rules for forming input map from next state map	
Type of F/F	Input	Q+ =0	Q+=1	Q+ = 0	Q+ = 1	Q=0 Half of Map	Q=1 Half of Map
D-F/F	D	0	1	0	1	No change	No change

Next State Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	x

Q+

D- F/F Input Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	x

D = Q’A’B’+QB’+AB’

Derivation of Flip-Flop Input Equations from Next State Maps

Type of F/F	Input	Q=0		Q=1		Rules for forming input map from next state map	
		Q+ =0	Q+=1	Q+ = 0	Q+ = 1	Q=0 Half of Map	Q=1 Half of Map
D-F/F	D	0	1	0	1	No change	No change
T-F/F	T	0	1	1	0	No change	Complement

Next State Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	x

Q+

T – F/F Input Map

AB	0	1
00	0	0
01	1	1
11	0	1
10	1	x

T = A’B+AB’+QB

Derivation of Flip-Flop Input Equations from Next State Maps

Type of F/F	Input	Q=0		Q=1		Rules for forming input map from next state map	
		Q+ =0	Q+=1	Q+ = 0	Q+ = 1	Q=0 Half of Map	Q=1 Half of Map
D-F/F	D	0	1	0	1	No change	No change
T-F/F	T	0	1	1	0	No change	Complement
S-R F/F	S	0	1	0	x	No change	Replace 1's with x's
	R	x	0	1	0	*Replace 0's with x's *Replace 1's with 0's	Complement

Next State Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	x

Q+

R-S F/F Input Maps

AB \ Q	0	1
00	0	x
01	1	0
11	0	0
10	1	x

S = AB’+Q’A’B

AB \ Q	0	1
00	x	0
01	0	1
11	x	1
10	0	x

R = QB

Derivation of Flip-Flop Input Equations from Next State Maps

Type of F/F	Input	Q=0: Q+ =0	Q=0: Q+=1	Q=1: Q+ = 0	Q=1: Q+ = 1	Rules for forming input map from next state map: Q=0 Half of Map	Rules for forming input map from next state map: Q=1 Half of Map
D-F/F	D	0	1	0	1	No change	No change
T-F/F	T	0	1	1	0	No change	Complement
R-S F/F	S	0	1	0	x	No change	Replace 1's with x's
	R	x	0	1	0	*Replace 0's with x's *Replace 1's with 0's	Complement
J-K F/F	J	0	1	x	x	No change	Fill in with x's
	K	x	x	1	0	Fill in with x's	Complement

Next State Map

AB \ Q	0	1
00	0	1
01	1	0
11	0	0
10	1	x

Q+

J-K F/F Input Maps

AB	0	1
00	0	x
01	1	x
11	0	x
10	1	x

J = A’B+AB’

AB	0	1
00	x	0
01	x	1
11	x	1
10	x	x

K = B

Next - State Maps

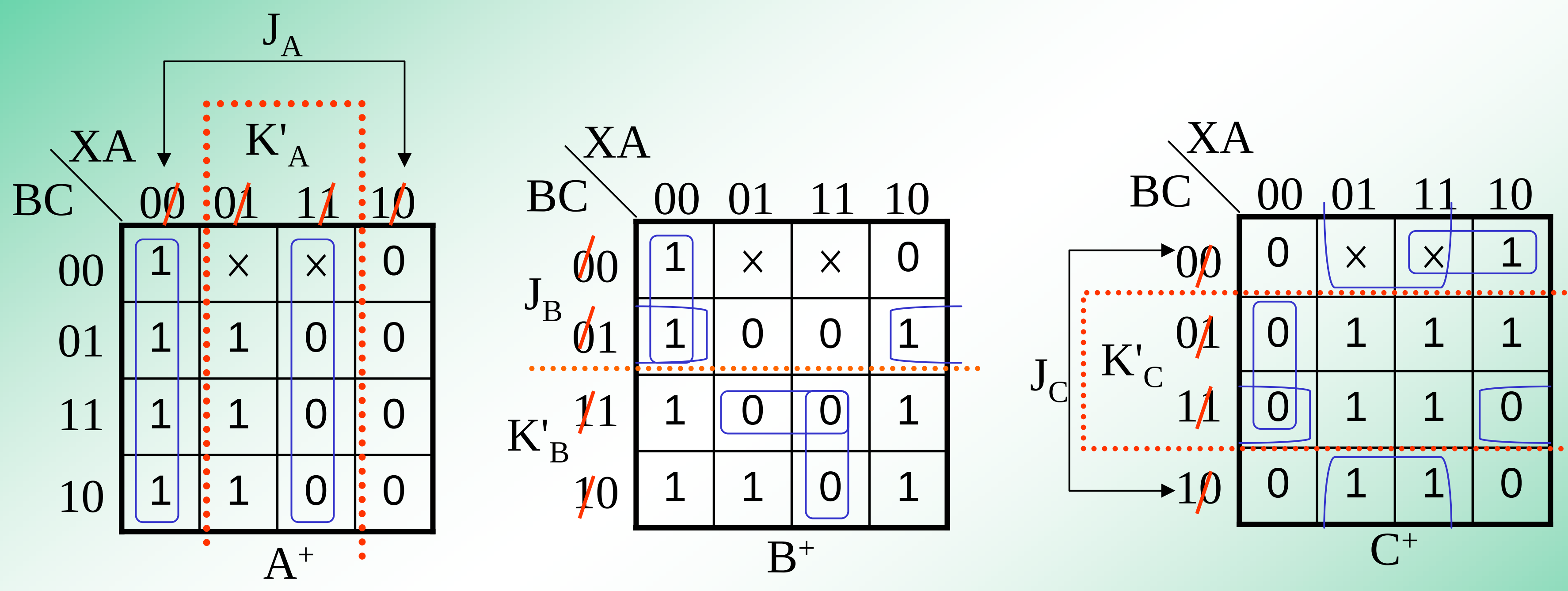
ABC	A+ X=0	A+ X=1	B+ X=0	B+ X=1	C+ X=0	C+ X=1
000	1	0	1	0	0	1
001	1	0	1	1	0	1
010	1	0	1	1	0	0
011	1	0	1	1	0	0
100	×	×	×	×	×	×
101	1	0	0	0	1	1
110	1	0	1	0	1	1
111	1	0	0	0	1	1

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	0	0	1
11	1	0	0	1
10	1	1	0	1

BC \ XA	00	01	11	10
00	0	×	×	1
01	0	1	1	1
11	0	1	1	0
10	0	1	1	0

Derivation of J-K Input Equations from Next State Maps



$J_A = X'$

$K_A = X$

$J_B = X'A' + A'C$

$K_B = AC + XA$

$J_C = A + XB'$

$K_C = X'A' + A'B$

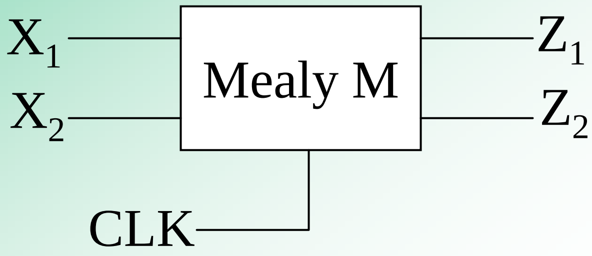
Derivation of J-k F/F input equations

(3) Choose S-R F/F

Q	Q^+	S	R
0	0	0	×
0	1	1	0
1	0	0	1
1	1	×	0

	$A^+B^+C^+$		S_AR_A		S_BR_B		S_CR_C	
	X=		X=		X=		X=	
ABC	0	1	0	1	0	1	0	1
000	110	001	1 0	0 x	:	:	:	:
110	111	001	x 0	0 1	:	:	:	:
001	110	011	1 0	0 x	:	:	:	:
111	101	001	x 0	0 1	:	:	:	:
011	110	010	1 0	0 x	:	:	:	:
101	101	001	x 0	0 1	:	:	:	:
010	110	010	1 0	0 x	:	:	:	:

Ex: Another example with 2 inputs X_1X_2 , 2 outputs Z_1Z_2



	N.S. x_1x_2=				Z x_1x_2=			
P.S.	00	01	11	10	00	01	11	10
S_0	S_0	S_0	S_1	S_1	00	00	01	01
S_1	S_1	S_3	S_2	S_1	00	10	10	00
S_2	S_3	S_3	S_2	S_2	11	11	00	00
S_3	S_0	S_3	S_2	S_0	00	00	00	00

4 states $\Rightarrow$ 2 F/F' s A , B

State Assignment : $S_0 = 00$, $S_1 = 01$, $S_2 = 11$, $S_3 = 10$

State transition Table

P.S.	N.S. $x_1x_2=$				Z $x_1x_2=$			
AB	00	01	11	10	00	01	11	10
00	00	00	01	01	00	00	01	01
01	01	10	11	01	00	10	10	00
11	10	10	11	11	11	11	00	00
10	00	10	11	00	00	00	00	00

D-F/F

BC \ XA	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	1	1
10	0	1	1	0

$$D_A = A^+ = X_2B + AB + X_2A$$

BC \ XA	00	01	11	10
00	0	0	1	1
01	1	0	1	1
11	0	0	1	1
10	0	0	1	0

$$D_B = B^+ = X_1A + X_2'A'B + X_1B + X_1X_2$$

BC \ XA	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	1	1	0	0
10	0	0	0	0

$$Z_1 = X_2A'B + X_1'AB$$

BC \ XA	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	1	1	0	0
10	0	0	0	0

$$Z_2 = X_1A'B' + X_1'AB$$

S-R F/F

Q	Q^+	S	R
0	0	0	×
0	1	1	0
1	0	0	1
1	1	×	0

	BC \ XA	00	01	11	10
A=0	00	0	0	0	0
A=0	01	0	1	1	0
A=1	11	×	×	×	×
A=1	10	0	×	×	0

$$S_A = X_2 B$$

BC \ XA	00	01	11	10
00	×	×	×	×
01	×	0	0	×
11	0	0	0	0
10	1	0	0	1

$$R_A = X_2'B'$$

	BC \ XA	00	01	11	10
B=0	00	0	0	1	1
B=1	01	0	0	×	×
B=1	11	×	0	×	×
B=0	10	0	0	1	0

$$S_B = X_1 X_2 + X_1 A'$$

BC \ XA	00	01	11	10
00	×	×	0	0
01	0	1	0	0
11	1	1	0	0
10	×	×	0	×

$$R_B = X_1' X_2 + X_1' A$$

Derivation of F/F Input Equations

State Graph

$\Rightarrow$ State Table

$\Rightarrow$ State Assignment

$\Rightarrow$ State Transition Table

$\Rightarrow$ K - map ← (Choice of F/F)

$\Rightarrow$ F/F Input Equations

Different state assignment
$\Rightarrow$ different F/F Input Equations !

15-7 Equivalent State Assignments

§ Cost of logic strongly depends on state assignments.

Ex:

Assign 1 AB	Assign 2 AB	P.S.	N.S. $x=0$	N.S. $x=1$	Z $x=0$	Z $x=1$
11	00	S_1	S_1	S_3	0	0
10	01	S_2	S_2	S_1	0	1
01	10	S_3	S_2	S_3	1	0

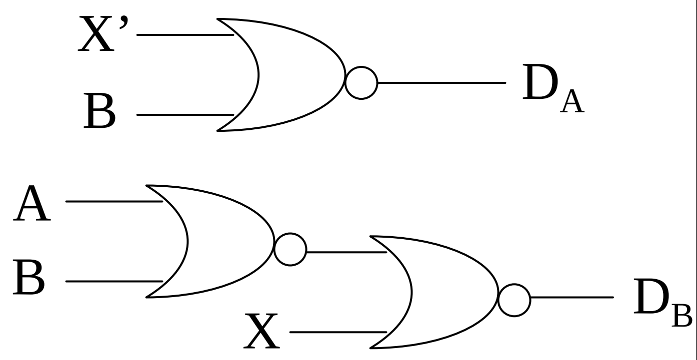
* Assignment 1

$S_1 = 00\,,\, S_2 = 01\,,\, S_3 = 10$

D - F/F : 2 F/F's A , B

$D_A = XB'$ $\quad D_B = X'(A+B)$

NOR implementation



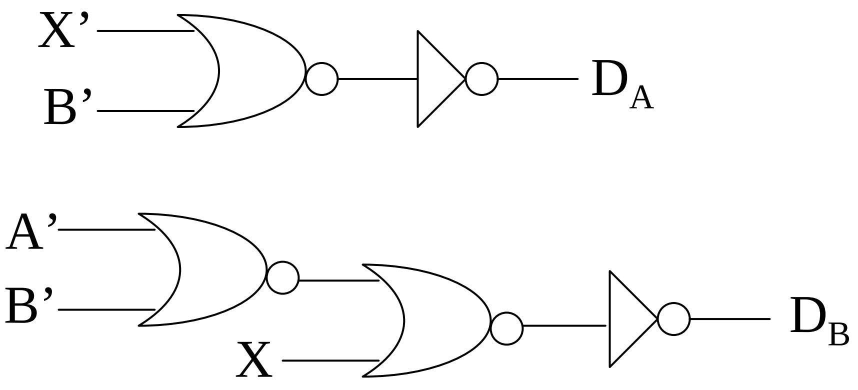
* Assignment 2

$$S_1 = 11, S_2 = 10, S_3 = 01$$

D - F/F : 2 F/F's A , B

$$D_A = X' + B' \qquad D_B = X + A'B'$$

NOR implementation



$\Rightarrow$ 2 more gates for NOR implementation

Adjacent states

2 states are adjacent

$$\left.\begin{array}{lcl} S_1(010) & \longleftrightarrow & S_2(011) \\ (100) & \longleftrightarrow & (110) \end{array}\right\}\text{adjacent}$$

$$\left.\begin{array}{lcl} S_1(010) & & S_2(111) \\ (001) & & (111) \end{array}\right\}\text{not adjacent}$$

15-8 Guildlines for State Assignment

work for D F/F , J - K F/F
not for T F/F , S - R F/F

1. States which have the same next state should be given adjacent assignment .

2. States which are next states of the same state should be given adjacent assignment .

3. States which have the same output
$\Rightarrow$ adjacent states

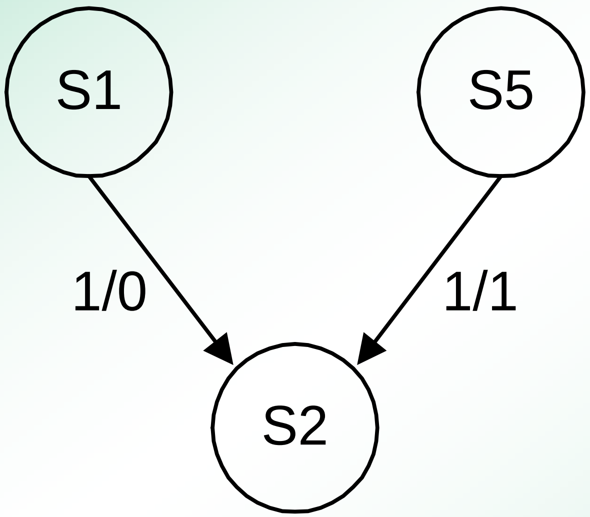
To minimize output function

In above order !!

No guarantee a minimum solution

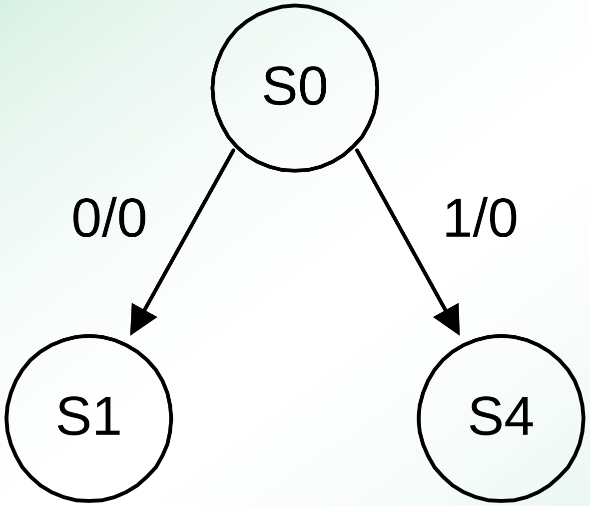
Guideline 1：**在相同的input條件下，會跳到同一個next state 的states。**

state has the same n. s.



根據guideline 1，S1、S5需要相鄰。

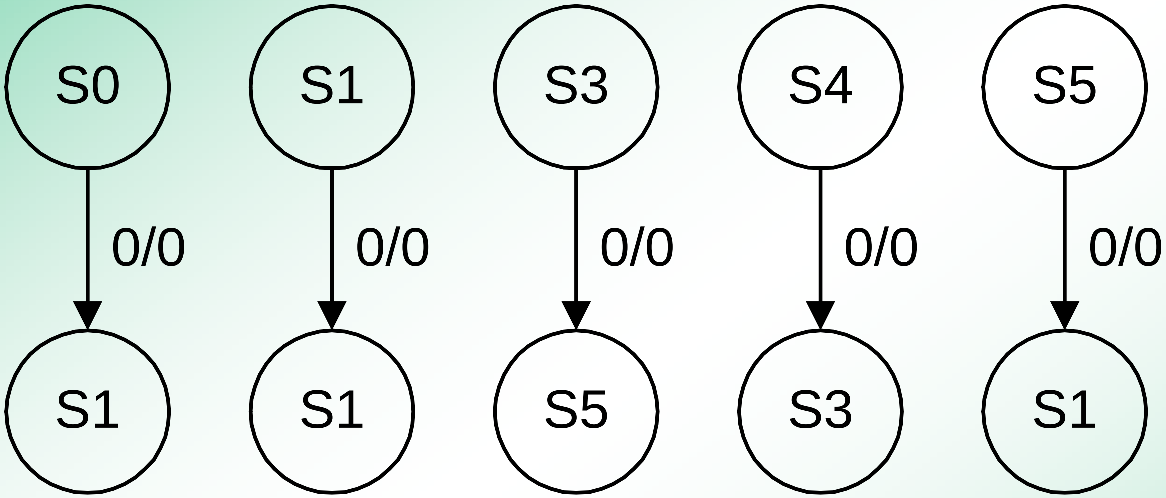
***Guideline 2*：同一個state跳出來的next states。**
states has the same p.s.



根據guideline 2，S1、S4需要相鄰。

Guideline 3：在相同的**input**條件下，會產生相同**output**的**states**。

same output states



根據guideline 3，S0、S1、S3、S4、S5需要相鄰。

Ex:

P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
S_0	S_1	S_2	0	0
S_1	S_3	S_2	0	0
S_2	S_1	S_4	0	0
S_3	S_5	S_2	0	0
S_4	S_1	S_6	0	0
S_5	S_5	S_2	1	0
S_6	S_1	S_6	0	1

3 F/F's A, B, C

1.list relationship

States having the same n.s.

Rule **1**: $(S_0, S_1, S_3, S_5)\ (S_3, S_5)\ (S_4, S_6)\ (S_0, S_2, S_4, S_6)$

Rule **2**: $(S_1, S_2)\ (S_2, S_3)\ (S_1, S_4)\ (S_2, S_5) \times \mathbf{2}\ \ (S_1, S_6) \times \mathbf{2}$

States having the same p.s.

2.Use an assignment map

BC \ A	0	1
00	S_0	
01	S_2	S_5
11	S_4	S_3
10	S_6	S_1

Assignment (a)

BC \ A	0	1
00	S_0	
01	S_1	S_6
11	S_3	S_4
10	S_5	S_2

Assignment (b)

States having the same n.s.

High priority

Rule **1**: $(S_0,S_1,S_3,S_5)\ (S_3,S_5)\ (S_4,S_6)\ (S_0,S_2,S_4,S_6)$

Rule **2**: $(S_1,S_2)\ (S_2,S_3)\ (S_1,S_4)\ (S_2,S_5)\times \mathbf{2}\ \ (S_1,S_6)\times \mathbf{2}$

States having the same p.s.

Assignment(a)

	P.S.	N.S. x = 0	N.S. x = 1	Z x = 0	Z x = 1
000	S_0	S_1	S_2	0	0
110	S_1	S_3	S_2	0	0
001	S_2	S_1	S_4	0	0
111	S_3	S_5	S_2	0	0
011	S_4	S_1	S_6	0	0
101	S_5	S_5	S_2	1	0
010	S_6	S_1	S_6	0	1

6 gates 13 literals

Another straight forward assignment

	P.S.	N.S. $x=0$	N.S. $x=1$	Z $x=0$	Z $x=1$
000	S_0	S_1	S_2	0	0
001	S_1	S_3	S_2	0	0
010	S_2	S_1	S_4	0	0
011	S_3	S_5	S_2	0	0
100	S_4	S_1	S_6	0	0
101	S_5	S_5	S_2	1	0
110	S_6	S_1	S_6	0	1

10 gates 39 literals

Why guideline help ?

ABC		$A^+B^+C^+$ x = 0		$A^+B^+C^+$ x = 1		Z x = 0	Z x = 1
000	S_0	110	S_1	001	S_2	0	0
110	S_1	111	S_3	001	S_2	0	0
001	S_2	110	S_1	011	S_4	0	0
111	S_3	101	S_5	001	S_2	0	0
011	S_4	110	S_1	010	S_6	0	0
101	S_5	101	S_5	001	S_0	1	0
010	S_6	110	S_1	010	S_6	0	1

Next state map

		$A^+B^+C^+$ x = 0		$A^+B^+C^+$ x = 1		Z x = 0	Z x = 1
S_0	000	110	S_1	001	S_2	0	***0***
S_2	001	110	S_1	011	S_4	0	***0***
S_6	010	110	S_1	010	S_6	0	***1***
S_4	011	110	S_1	010	S_6	0	***0***
–	100	×××		×××		–	–
S_5	101	***101***	S_5	***001***	S_2	***1***	***0***
S_1	110	***111***	S_3	***001***	S_2	***0***	***0***
S_3	111	***101***	S_5	***001***	S_2	***0***	***0***

State transition graph

Next state map

BC \ XA	00	01	11	10
00	S_1 (S_0)	×	×	S_2 (S_0)
01	S_1 (S_2)	S_5 (S_5)	S_2 (S_5)	S_4 (S_2)
11	S_1 (S_4)	S_5 (S_3)	S_2 (S_3)	S_6 (S_4)
10	S_1 (S_6)	S_3 (S_1)	S_2 (S_1)	S_6 (S_6)

$S_0\ S_2\ S_4\ S_6$

$S_3\ S_5$

$S_1\ S_3\ S_5$

$S_4\ S_6$

A^+

BC \ XA	00	01	11	10
00	1	×	×	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

$A^+ = X'$

B^+

BC \ XA	00	01	11	10
00	1	×	×	1
01	1	0	0	1
11	1	0	0	1
10	1	1	0	1

C^+

BC \ XA	00	01	11	10
00	0	×	×	1
01	0	1	1	1
11	0	1	1	0
10	0	1	1	0

due to $S_2\ S_5$ assignment

Ex:

	N.S.		Z	
P.S.	x = 0	x = 1	x = 0	x = 1
a	a	c	0	0
b	d	f	0	1
c	c	a	0	0
d	d	b	0	1
e	b	f	1	0
f	c	e	1	0

Guidelines

1.*(b,d) (c,f) (b,e)*

2.*(a,c)*×***2*** *, (d,f) (b,d)*

(b,f) (c,e)

3.*(a,c) (b,d) (e,f)*

Assignment (a)

Q_2Q_3 \ Q_1	0	1
00	a	c
01		e
11	b	d
10		f

a = 000

b = 111

c = 100

d = 011

e = 101

f = 110

Assignment (b)

Q_2Q_3 \ Q_1	0	1
00	c	a
01		e
11	d	b
10	f	

a = 100

b = 111

c = 000

d = 011

e = 101

f = 010

1.(b,d) (c,f) (b,e)

2.(a,c)×**2** , (d,f) (b,d)

(b,f) (c,e)

3.(a,c) (b,d) (e,f)

For Assignment (b)

	P.S. $Q_1Q_2Q_3$	N.S. $x=0$	N.S. $x=1$	Z $x=0$	Z $x=1$
m_4	100	100	000	0	0
m_7	111	011	010	0	1
m_0	000	000	100	0	0
m_3	011	011	111	0	1
m_5	101	111	010	1	0
m_2	010	000	101	1	0
m_1	001	– – –	– – –	–	–
m_6	110	– – –	– – –	–	–

State Transition Table

	P.S. $Q_1Q_2Q_3$	N.S. $x=0$	N.S. $x=1$	Z $x=0$	Z $x=1$
m_0	000	000	100	0	0
m_1	001	– – –	– – –	–	1
m_2	010	000	101	1	0
m_3	011	011	111	0	1
m_4	100	100	000	0	0
m_5	101	111	010	1	0
m_6	110	– – –	– – –	–	–
m_7	111	***011***	***010***	***0***	***1***

next state map

Q_2Q_3 \ XQ_1	00	01	11	10
00	000	100	000	100
01	×	111	010	×
11	011	011	010	111
10	000	×	×	101

Next State Map

J_1: $Q_1=0$ (columns 00, 10)

Q_2Q_3 \ XQ_1	00	01	11	10
00	0	1	0	0
01	×	1	0	×
11	0	0	0	1
10	0	×	×	1

$Q_1=1$, K_1' (columns 01, 11)

Q_1^+

$J_1 = XQ_2$

$k_1 = X + Q_2$

$Q_2=0$, J_2 (rows 00, 01)

$Q_2=1$, K_2' (rows 11, 10)

Q_2Q_3 \ XQ_1	00	01	11	10
00	0	1	0	0
01	×	1	0	×
11	0	0	0	1
10	0	×	×	1

Q_1^+

$J_2 = Q_3$

$k_2 = Q_3'$

$Q_3=1$, K_3' (rows 01, 11)

Q_2Q_3 \ XQ_1	00	01	11	10
00	0	1	0	0
01	×	1	0	×
11	0	0	0	1
10	0	×	×	1

$Q_3=0$, J_3 (rows 00, 10)

Q_3^+

$J_3 = XQ_2$

$k_3 = XQ_1$

D - F/F Implementation

Q_2Q_3 \ XQ_1	00	01	11	10
00		1		1
01	×	1		×
11				1
10		×	×	1

$$Q_1^+ = X'Q_1Q_2' + XQ_2'$$

Q_2Q_3 \ XQ_1	00	01	11	10
00				
01	×	1	1	×
11	1	1	1	1
10		×	×	

$$Q_2^+ = Q_3$$

Q_2Q_3 \ XQ_1	00	01	11	10
00				
01	×	1		×
11	1	1		1
10		×	×	1

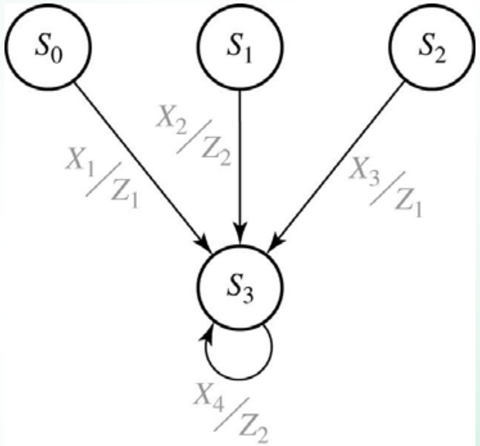
$$Q_3^+ = XQ_1'Q_3 + X'Q_2Q_3'$$

Q_2Q_3 \ XQ_1	00	01	11	10
00				
01	×	1		×
11			1	1
10	1	×	×	

$$Z = XQ_2Q_3 + X'Q_2'Q_3 + X'Q_2Q_3'$$

15-9 Using a One-Hot State Assignment

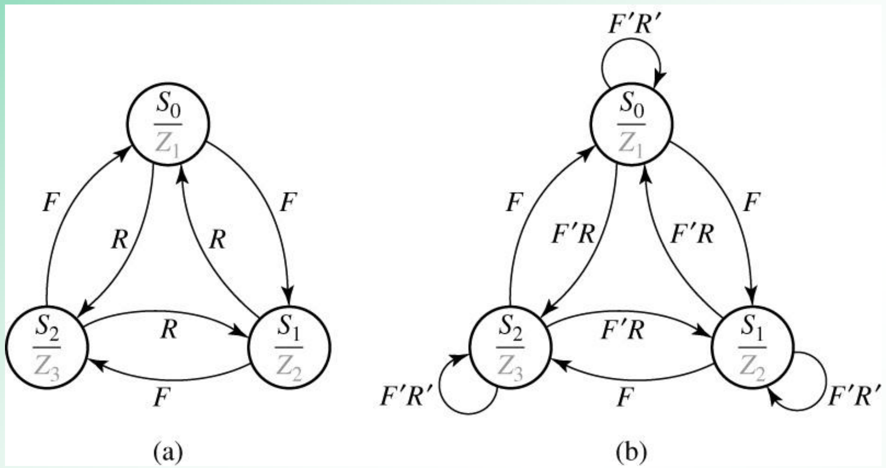
- When design with CPLD and FPGAs, it may not be important to minimize the number of F/F used in the design
- In order to design faster logic, the number of cells and interconnections between cells should be reduced
- **One-hot state assignment**
  - Using one F/F only for each state
  - 4 states use 4 F/F (Q_0, Q_1 , Q_2 , Q_3) with following state assignment
    S_0: $Q_0Q_1Q_2Q_3$=1000, **S_1**: 0100, **S_2**: 0010, **S_3**: 0001



Write next-state and output eq. directly by inspecting the state graph

$$Q_3^+ = X_1Q_0 + X_2Q_1 + X_3Q_2 + X_4Q_3$$

- For Moore machine,
  S₀: $Q_0Q_1Q_2$=100, **S_1**: 010, **S_2**: 001



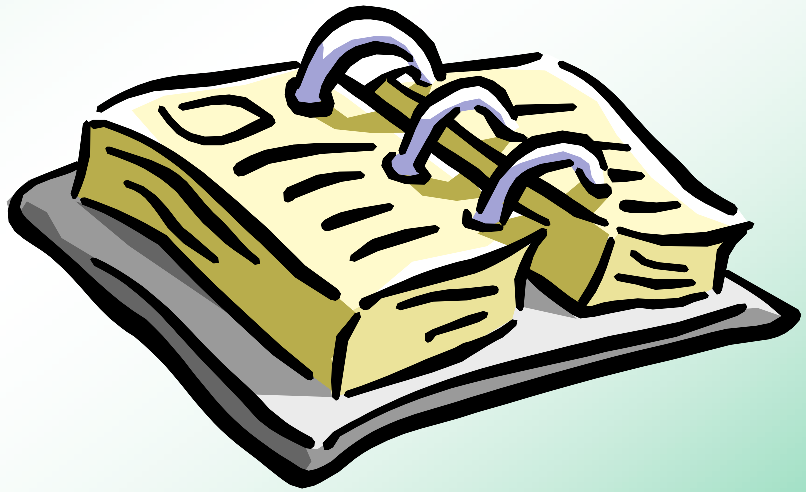
$$Q_1^+ = F'R'Q_0 + FQ_2 + F'RQ_1$$
$$Q_2^+ = F'R'Q_1 + FQ_0 + F'RQ_2$$
$$Q_3^+ = F'R'Q_2 + FQ_1 + F'RQ_0$$

$$Z_1 = Q_0 \quad Z_2 = Q_1 \quad Z_3 = Q_2$$

- When design with CPLDs or FPGAs, try both an assignment with a minimum number of state variables and a one-hot assignment

HOMEWORK -- Unit 15

- 15. 8
- 15. 20
- 15. 28



A Short Cut Method to Find J-K Flip-Flop Input Equations

$\because$ For J-K F/F : J_A, K_A independent of Q_A 又 $Q^+ = JQ' + K'Q$

$\Rightarrow$ $Q = 0$時 $Q^+ = J$ $Q = 1$時 $Q^+ = K'$

Q	Q^+	J	K
0	0	0	×
0	1	1	×
1	0	×	1
1	1	×	0

Example

Next state map : Q_A^+ Q_B^+ Q_C^+

P.S. $Q_AQ_BQ_C$	N. S. $Q_A^+Q_B^+Q_C^+$
000	111
001	xxx
010	000
011	000
100	111
101	xxx
110	xxx
111	000

Q_A^+:

Q_BQ_C \ Q_A	0	1
00	1	1
01	×	×
11	0	0
10	0	×

Q_B^+:

Q_BQ_C \ Q_A	0	1
00	1	1
01	×	×
11	0	0
10	0	×

Q_C^+:

Q_BQ_C \ Q_A	0	1
00	1	1
01	×	×
11	0	0
10	0	×

$\because$ For J - K F/F : J_A , K_A independent of Q_A 又 $Q^+ = JQ' + K'Q$

$\Rightarrow$ $Q = 0$時 $Q^+ = J$ $Q = 1$時 $Q^+ = K'$

Next state map : Q_A^+ Q_B^+ Q_C^+

Q_A+ (left column: J_A map, Q_A = 0; right column: K'_A map, Q_A = 1)

Q_BQ_C \ Q_A	0	1
00	1	1
01	×	×
11	0	0
10	0	×

$J_A = B'$ $K_A = B$ or C

Q_B+ (rows 00 and 01, B=0: J_B map; rows 11 and 10, B=1: K'_B map)

Q_BQ_C \ Q_A	0	1
00	1	1
01	×	×
11	0	0
10	0	×

$J_B=1$ $K_B=1$

Q_C+ (rows 00 and 10, C=0: J_C map; rows 01 and 11, C=1: K'_C map)

Q_BQ_C \ Q_A	0	1
00	1	1
01	×	×
11	0	0
10	0	×

$J_C=B'$ $K_C=1$