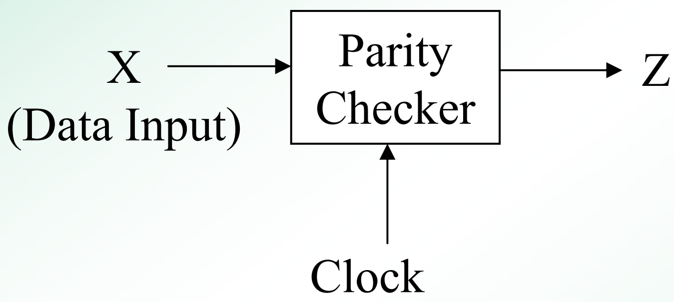
Chap. 13:

Analysis of Clocked Sequential Circuits

Deal with Sequential circuits which have inputs

13-1 Sequential Parity Checker

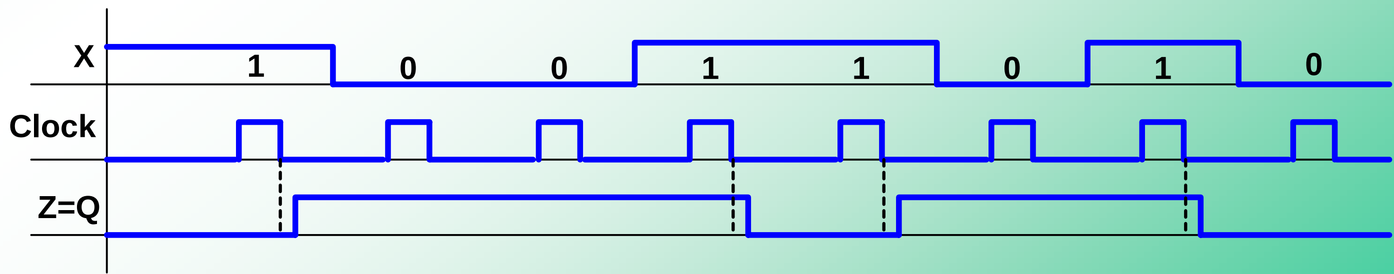
Parity bit :	odd parity	even parity
	0 0 0 0 0 0 0 1	*0 0 0 0 0 0 0 0*
	0 0 1 0 0 0 0 0	*0 0 1 0 0 0 0 1*
	1 0 0 1 1 0 1 1	*1 0 0 1 1 0 1 0*



X synchronous with Clock,

Input number of 1 odd $\Rightarrow Z = 1$

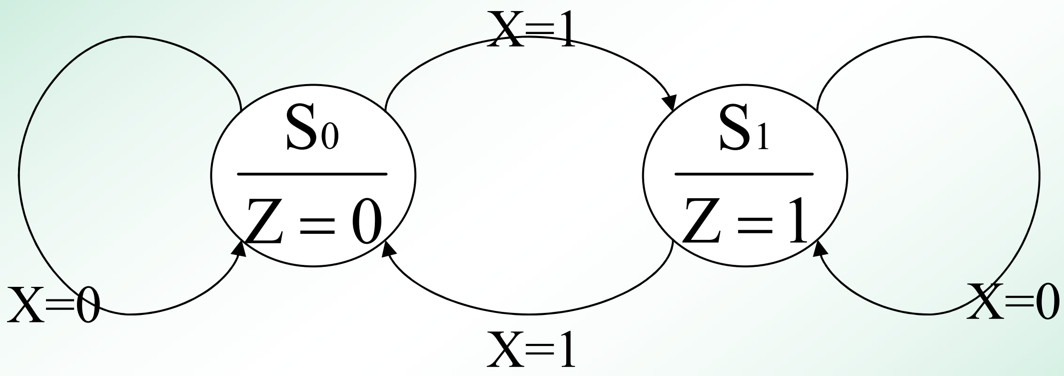
even $\Rightarrow Z = 0$



2 states required S_0 : even number of 1 received

S_1 : odd number of 1 received

Construct state graph ⇒



State Table

Present state	Next state X=0	Next state X=1	Present Output Z
S_0	S_0	S_1	0
S_1	S_1	S_0	1

Q	Q^+ X=0	Q^+ X=1	T X=0	T X=1	Z
0	0	1	0	1	0
1	1	0	0	1	1

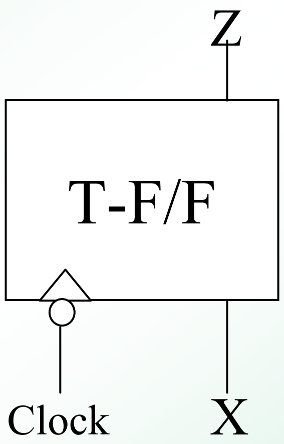
Use T-F/F to implement

	Q^+		T		
Q	X=0	X=1	X=0	X=1	Z
0	0	1	0	1	0
1	1	0	0	1	1

Q \ X	0	1
0	0	1
1	0	1

T

$T = X$



Use D-F/F to implement

Q	Q^+ X=0	Q^+ X=1	D X=0	D X=1	Z
0	0	1	0	1	0
1	1	0	1	0	1

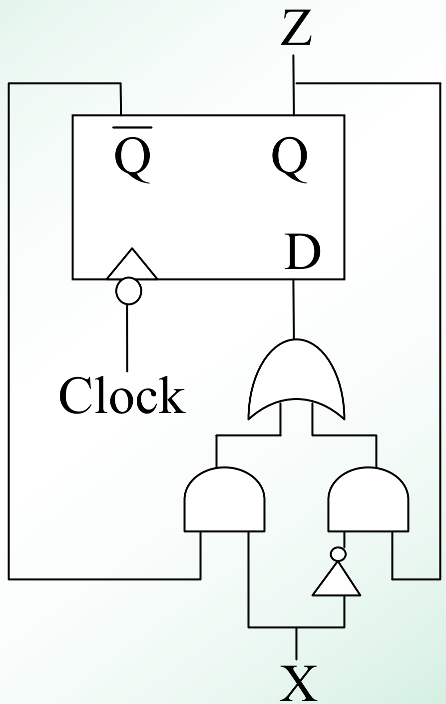
Q \ X	0	1
0	0	1
1	1	0

D

Q	
0	0
1	1

Z

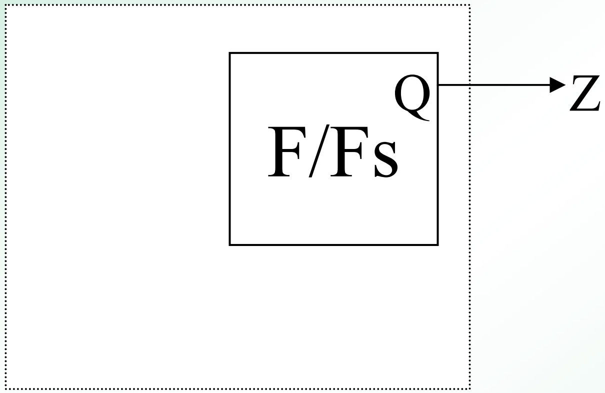
$$D = Q\overline{X} + \overline{Q}X \qquad Z = Q$$

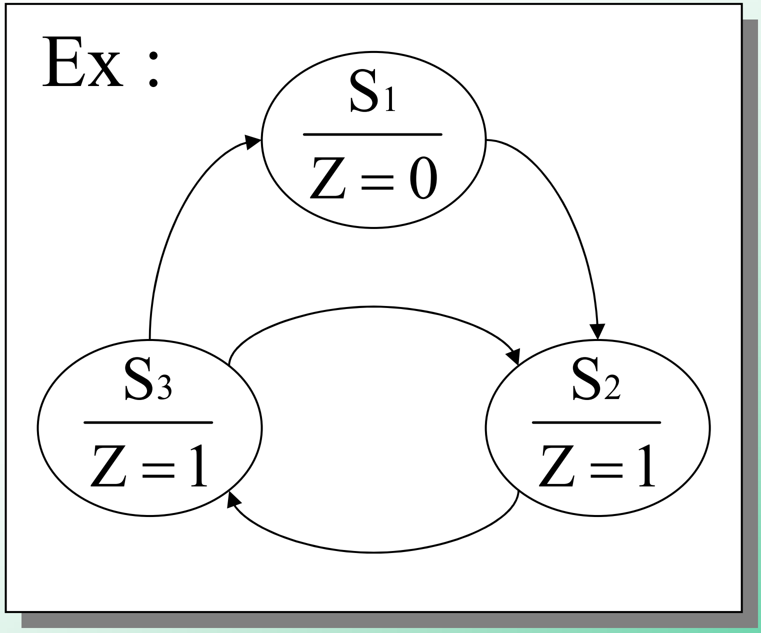


13-2 Analysis by Signal Tracing and Timing Charts

Ⅰ. Moore Machine : Outputs are function of the present state only.

State determined $\Rightarrow$ Output determined



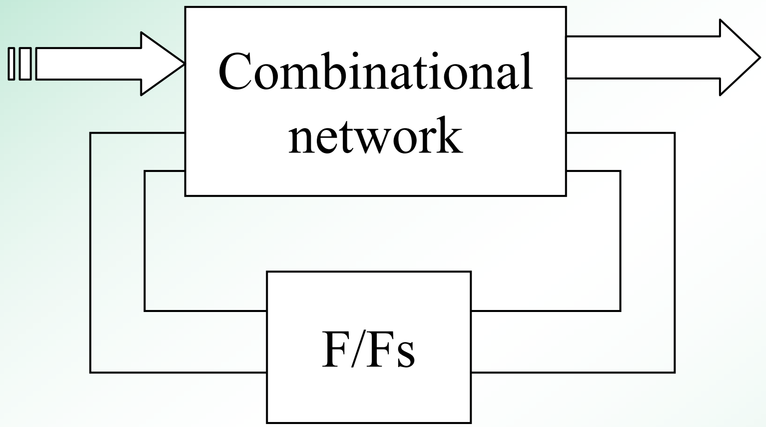


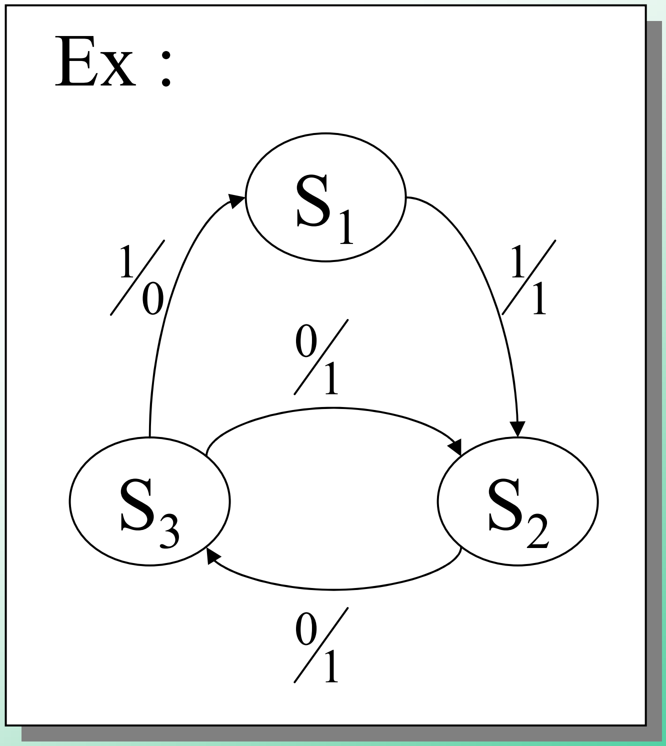
Outputs are state variables.

Outputs are associated with states.

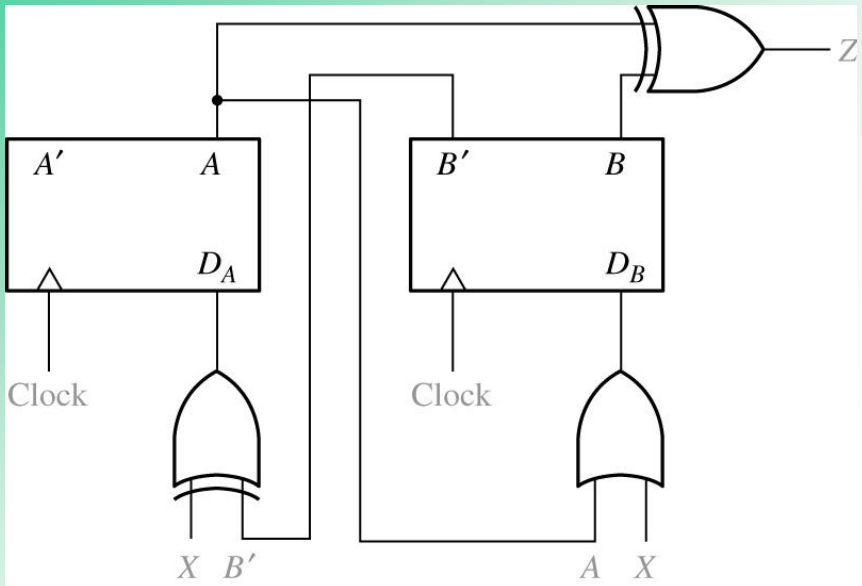
Ⅱ. Mealy Machine:

Outputs are functions of both the present states and the input.



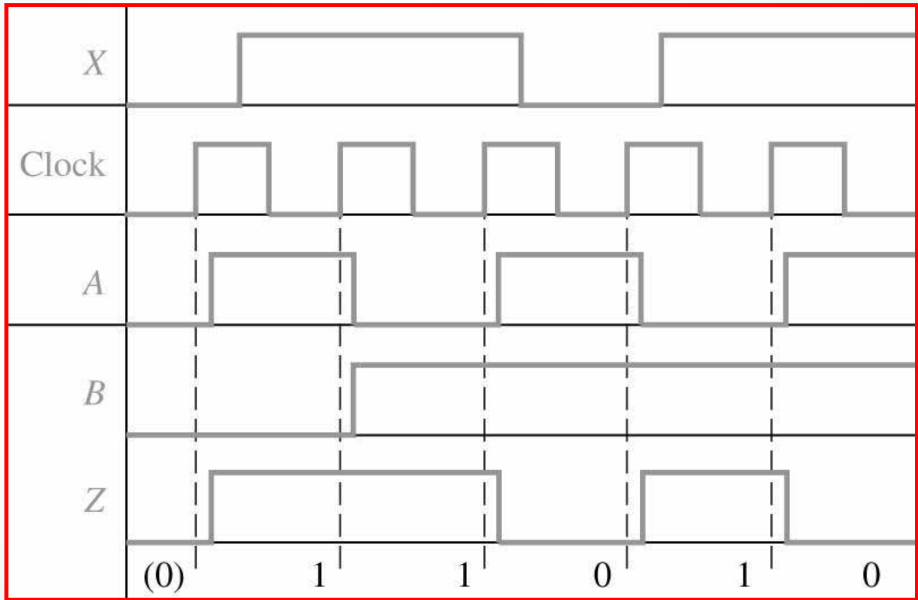


Analysis of Moore Machine : Output changes only after the clock pulse.

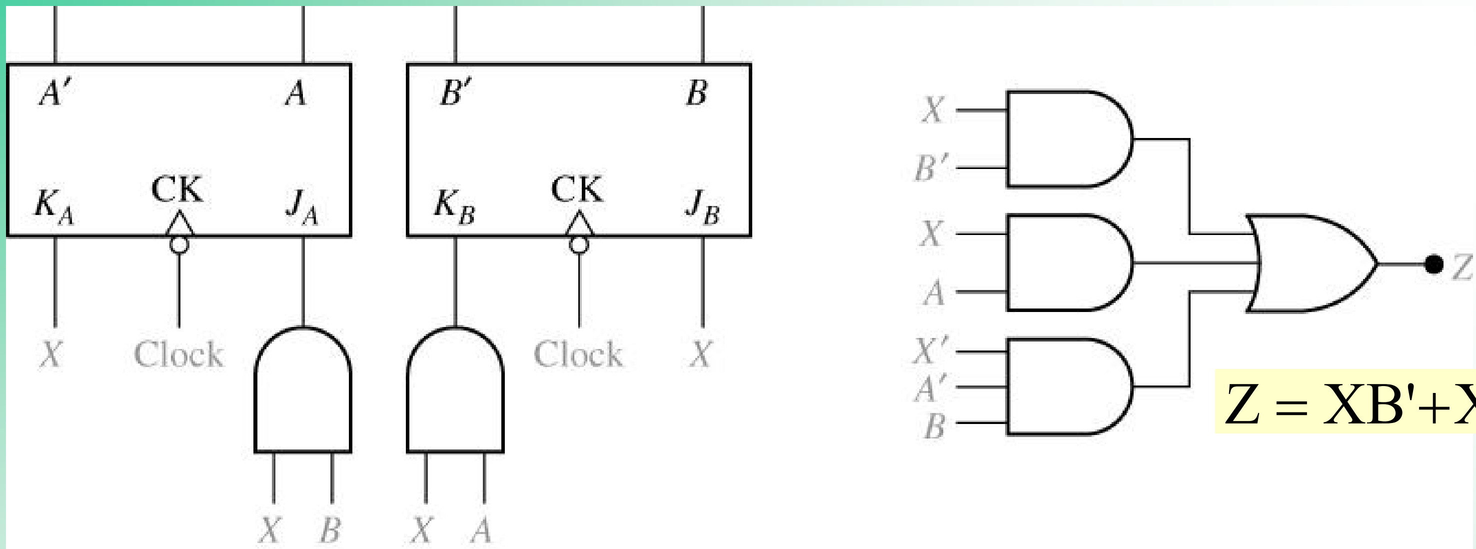


A , B initial state : 0 , 0 (reset)

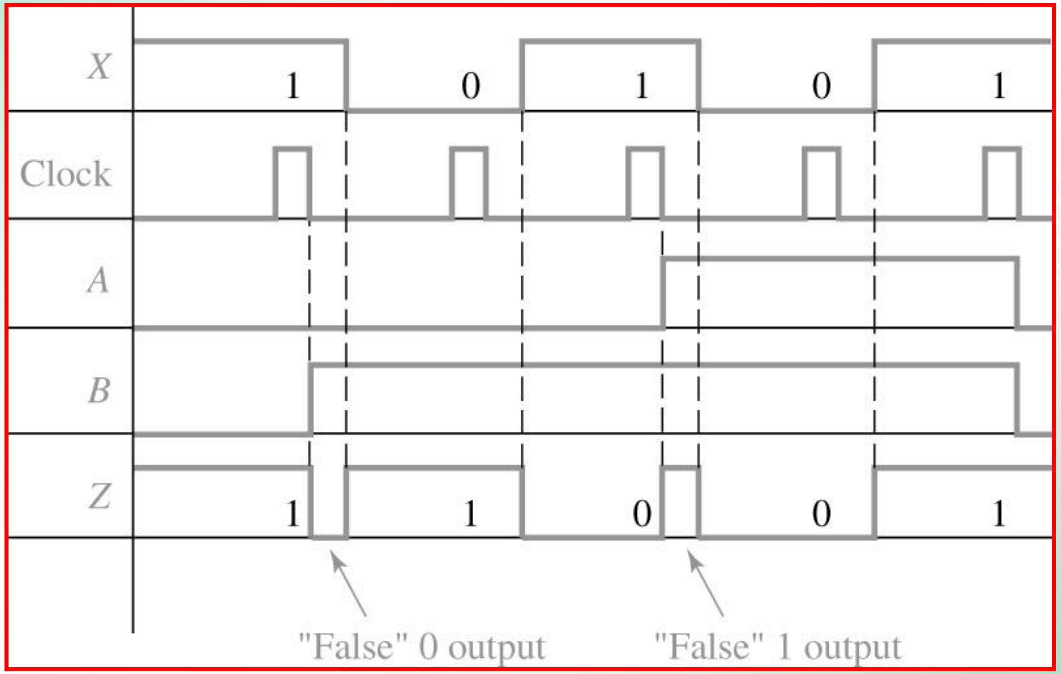
X	=	0	1	1	0	1	
A	=	0	1	0	1	0	1
B	=	0	0	1	1	1	1
Z	=	(0)	1	1	0	1	0



Analysis of Mealy Machine : Output changes only after the clock pulse.



$Z = XB' + XA + X'A'B$



A , B initial state : 0 , 0 (reset)

X	=	1	0	1	0	1	
A	=	0	0	0	1	1	0
B	=	0	1	1	1	1	0
Z	=	1(0)	1	0(1)	0	1	

13-3 State Tables and Graphs

Example:

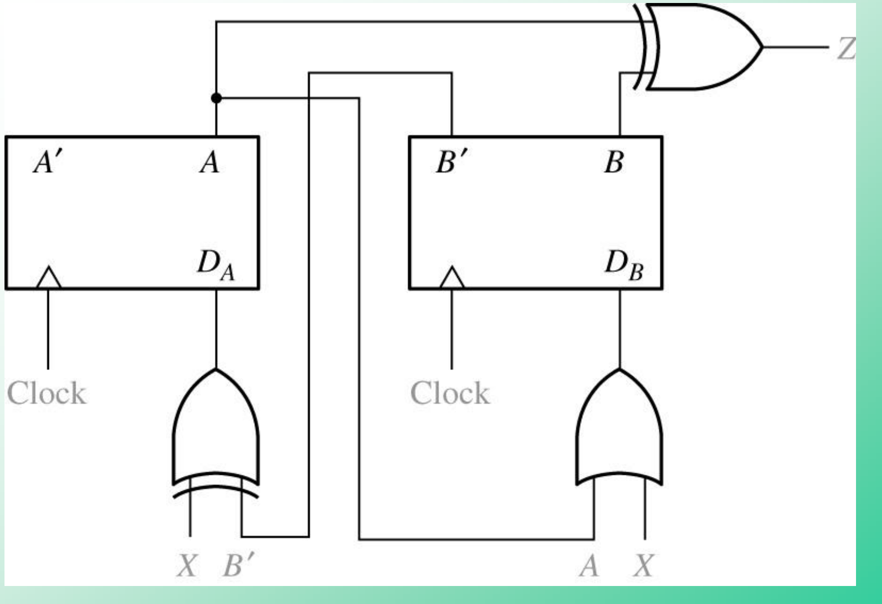
Construct state tables and graphs from logic circuits.

Moore machine

1. Determine the F/F input equation & the circuit output equation

$D_A = X \oplus B' \quad D_B = X + B$

$Z = A \oplus B$



2. Derive the next-state equations

$$A^{+} = D_A = X \oplus B'$$

$$B^{+} = D_B = X + A$$

3. Plot a next-state map

AB \ X	0	1
00	1	0
01	0	1
11	0	1
10	1	0

A^{+}

AB \ X	0	1
00	0	1
01	0	1
11	1	1
10	1	1

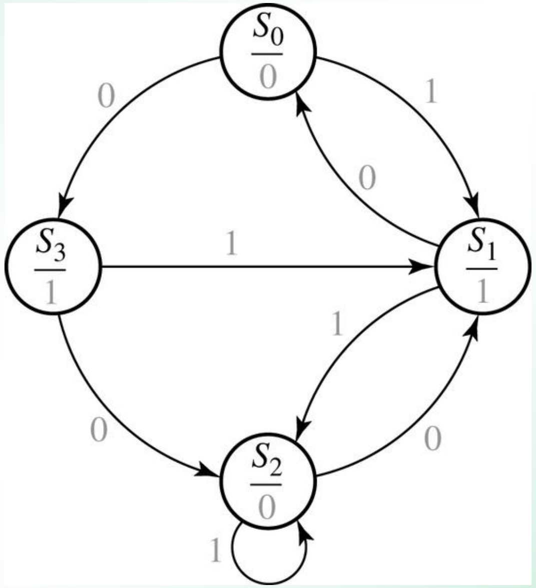
B^{+}

4. Combine all next-state maps to form the state table

AB	A^+B^+ X=0	A^+B^+ X=1	Z
00	10	01	0
01	00	11	1
11	01	11	0
10	11	01	1

Present state	Next state X=0	Next state X=1	Present Output (Z)
S_0	S_3	S_1	0
S_1	S_0	S_2	1
S_2	S_1	S_2	0
S_3	S_3	S_1	0

5. Corresponding state graph (Moore)



AB \ X	0	1
00	1	0
01	0	1
11	0	1
10	1	0

A^+

AB \ X	0	1
00	0	1
01	0	1
11	1	1
10	1	1

B^+

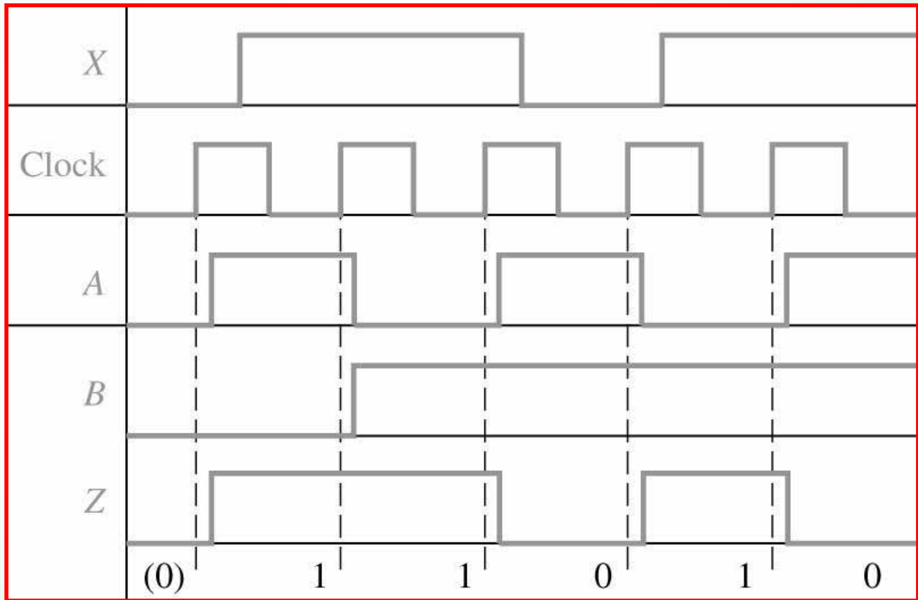
6. Construction of timing chart

Present state		Next state X=0	Next state X=1	Present Output (Z)
S_0	00	S_3	S_1	0
S_1	01	S_0	S_2	1
S_2	11	S_1	S_2	0
S_3	10	S_2	S_1	1

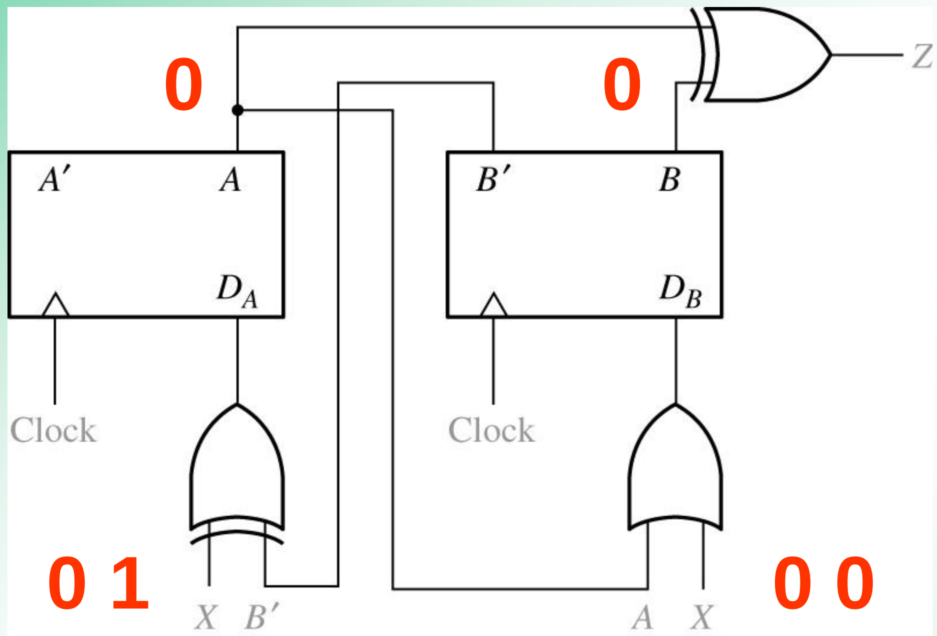
A , B initial state : 0 , 0 (reset)

X	=	0	1	1	0	1	
A	=	0	1	0	1	0	1
B	=	0	0	1	1	1	1
Z	=	(0)	1	1	0	1	0

Output changes only after the clock pulse.

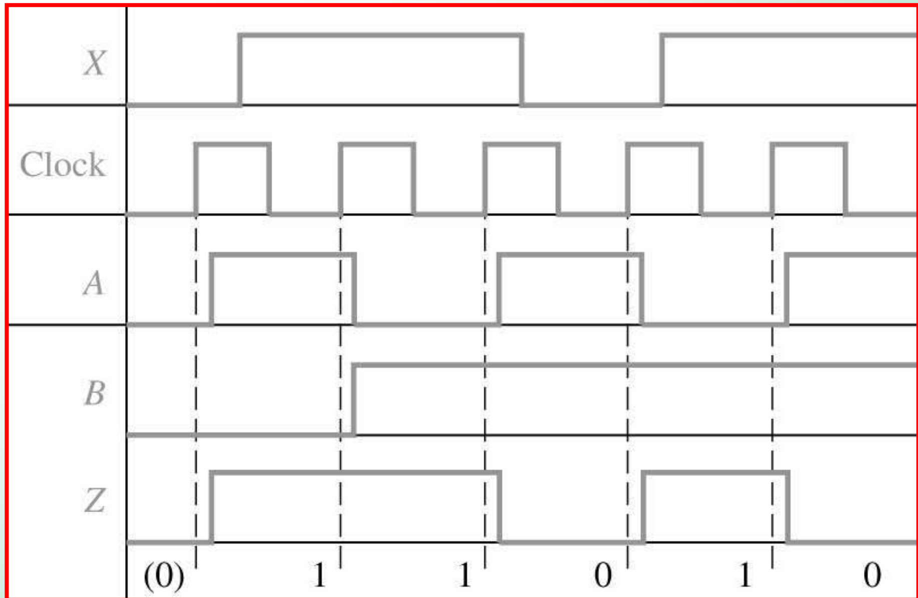


Analysis of Moore Machine

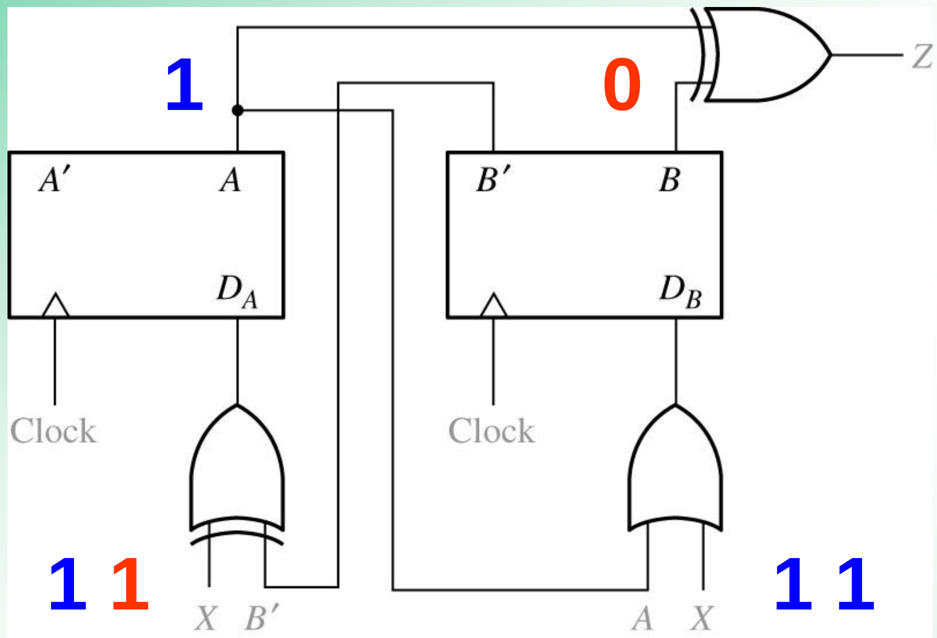


A , B initial state : 0 , 0 (reset)

X	=	0	1	1	0	1	
A	=	0	1	0	1	0	1
B	=	0	0	1	1	1	1
Z	=	(0)	1	1	0	1	0

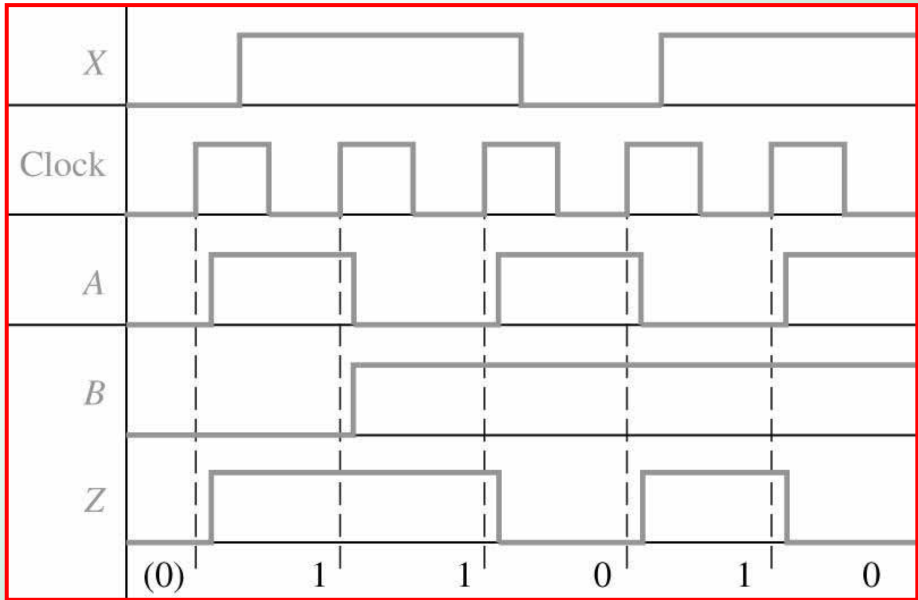


Analysis of Moore Machine

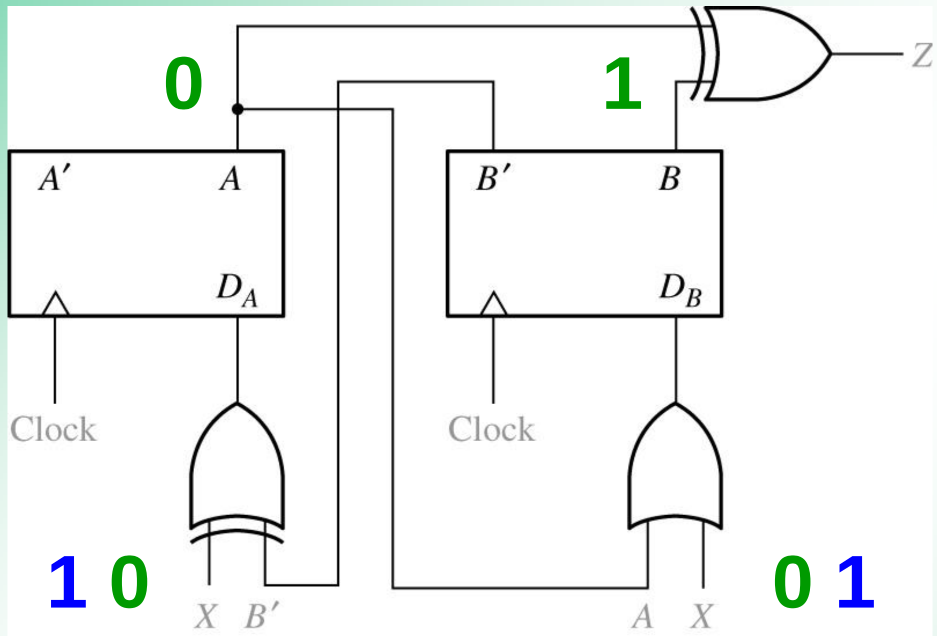


A , B initial state : 0 , 0 (reset)

X	=	0	1	1	0	1		
A	=	0	1	0	1	0	1	
B	=	0	0	1	1	1	1	
Z	=	(0)	1	1	0	1	0	

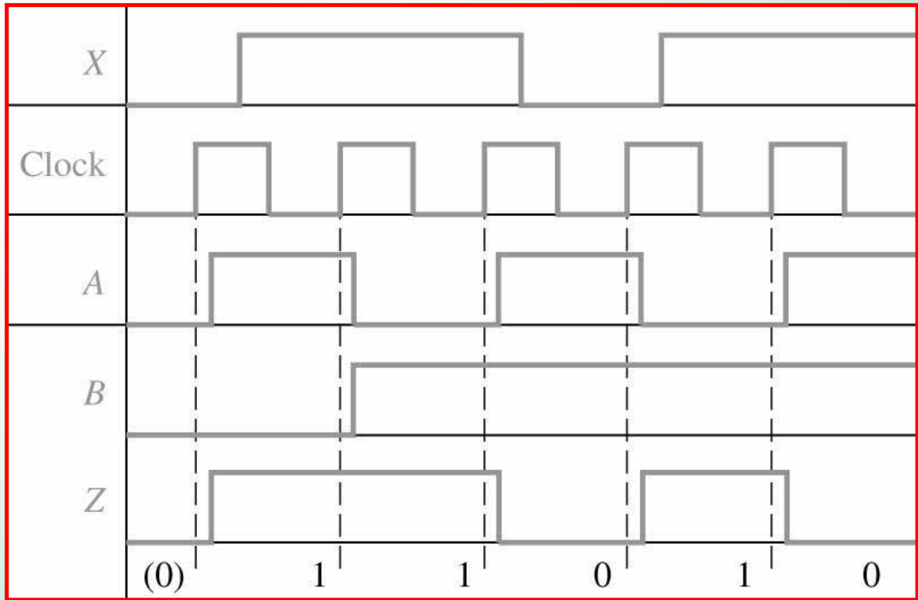


Analysis of Moore Machine

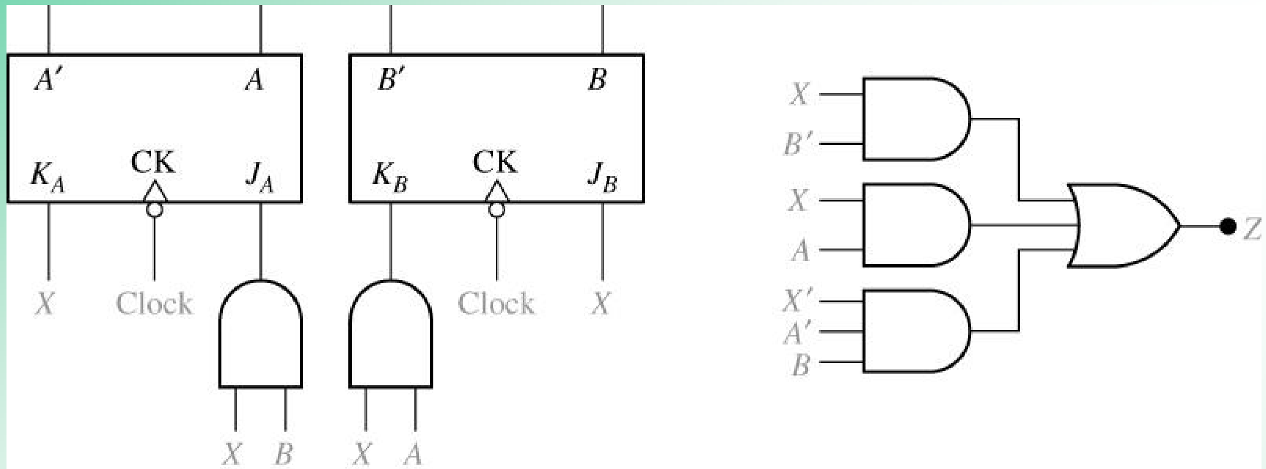


A , B initial state : 0 , 0 (reset)

X	=	0	1	1	0	1	
A	=	0	1	0	1	0	1
B	=	0	0	1	1	1	1
Z	=	(0)	1	1	0	1	0



Mealy machine



1. Determine the F/F input equation & the circuit output equation

$$\left\{ \begin{array}{l} J_A = XB, K_A = X \\ J_B = X, K_B = XA \end{array} \right\}$$

$$\mathbf{Z = X'A'B + XB' + XA}$$

2. Derive the next-state equations

$$\mathbf{A^+ = J_A A' + K_A' A = XBA' + X'A}$$

$$\mathbf{B^+ = J_B B' + K_B' B = XB' + (A'X')B = XB' + X'B + A'B}$$

3. Plot a next-state map

A^+

AB \ X	0	1
00	0	0
01	0	1
11	1	0
10	1	0

B^+

AB \ X	0	1
00	0	1
01	1	1
11	1	0
10	0	1

Z

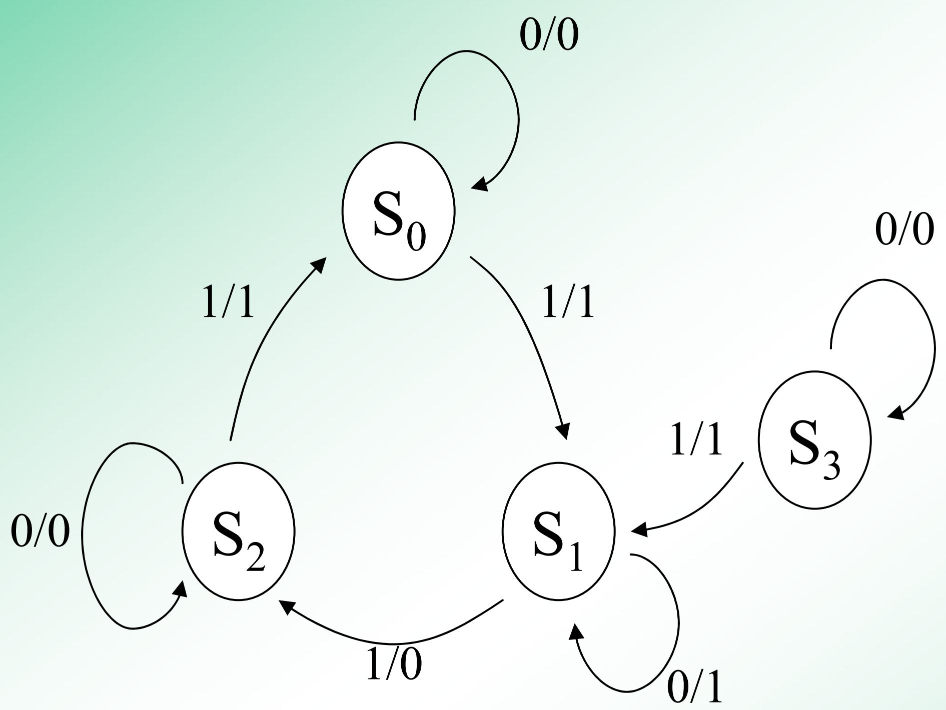
AB \ X	0	1
00	0	1
01	1	0
11	0	1
10	0	1

4. Combine all next-state maps to form the state table

AB	A^+B^+		Z	
	X=0	X=1	X=0	X=1
00	00	01	0	1
01	01	11	1	0
11	11	00	0	1
10	10	01	0	1

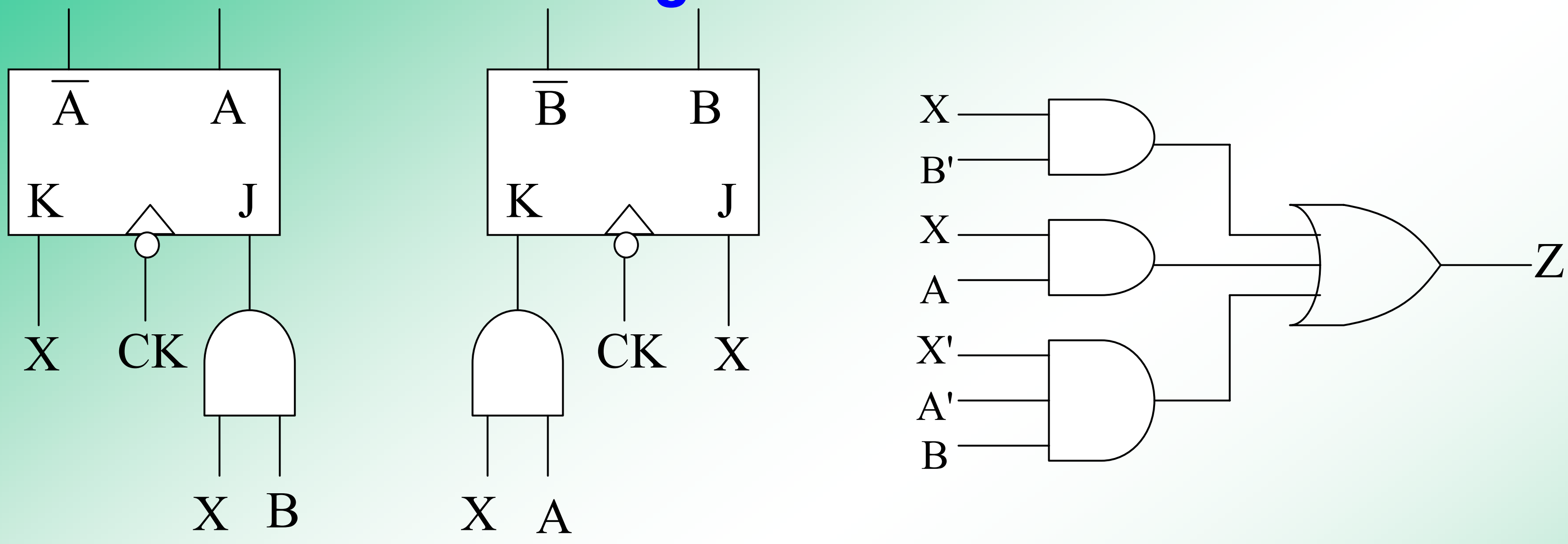
Present state	Next state		Present Output (Z)	
	X=0	X=1	X=0	X=1
S_0	S_0	S_1	0	1
S_1	S_1	S_2	1	0
S_2	S_2	S_0	0	1
S_3	S_3	S_1	0	1

5. Corresponding state graph (Mealy)



Present state	Next state X=0	Next state X=1	Present Output (Z) X=0	Present Output (Z) X=1
S_0	S_0	S_1	0	1
S_1	S_1	S_2	1	0
S_2	S_2	S_0	0	1
S_3	S_3	S_1	0	1

6. Construction of timing chart

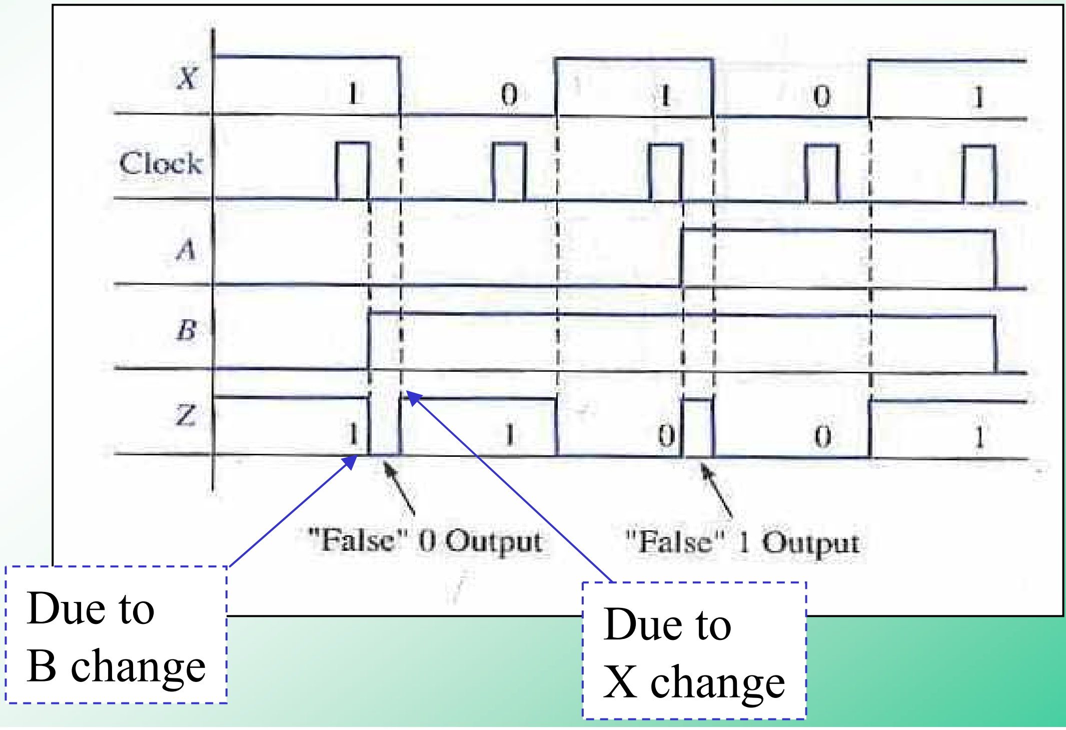


$X = 1\,0\,1\,0\,1$

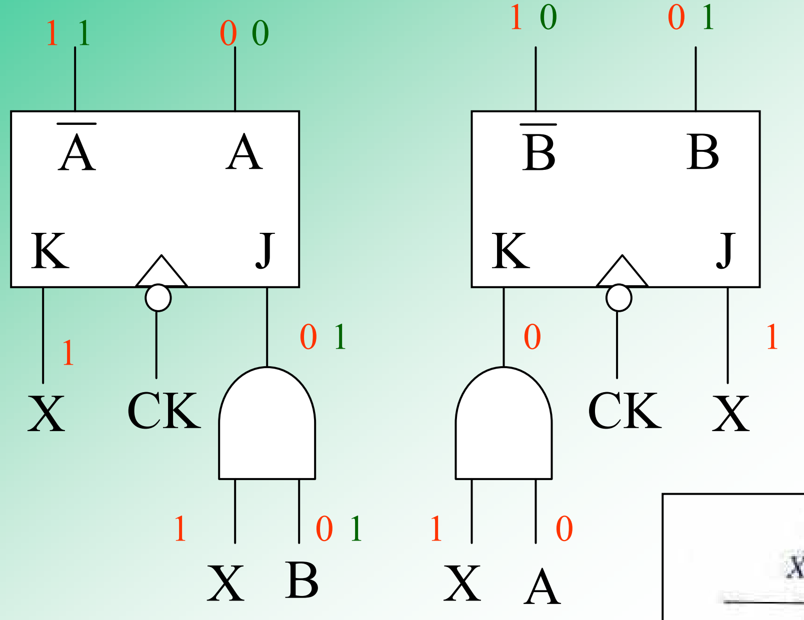
$A = 0\,0\,0\,1\,1\,0$

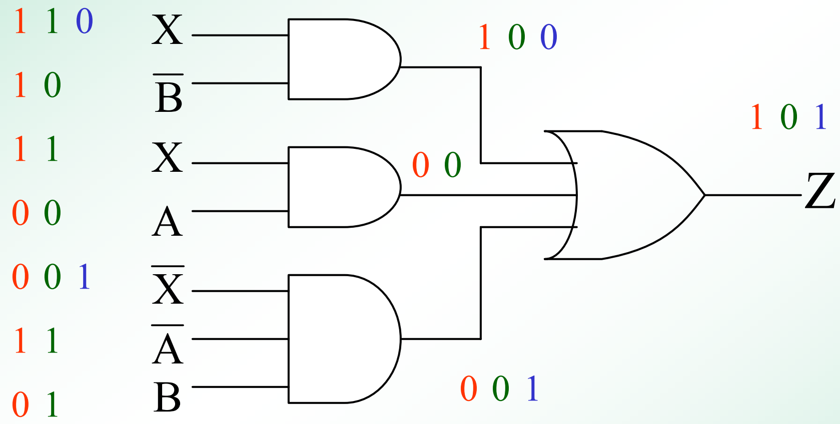
$B = 0\,1\,1\,1\,1\,0$

$Z = 1\,(0)\,1\,0\,(1)\,0\,1$



Analysis of Mealy Machine



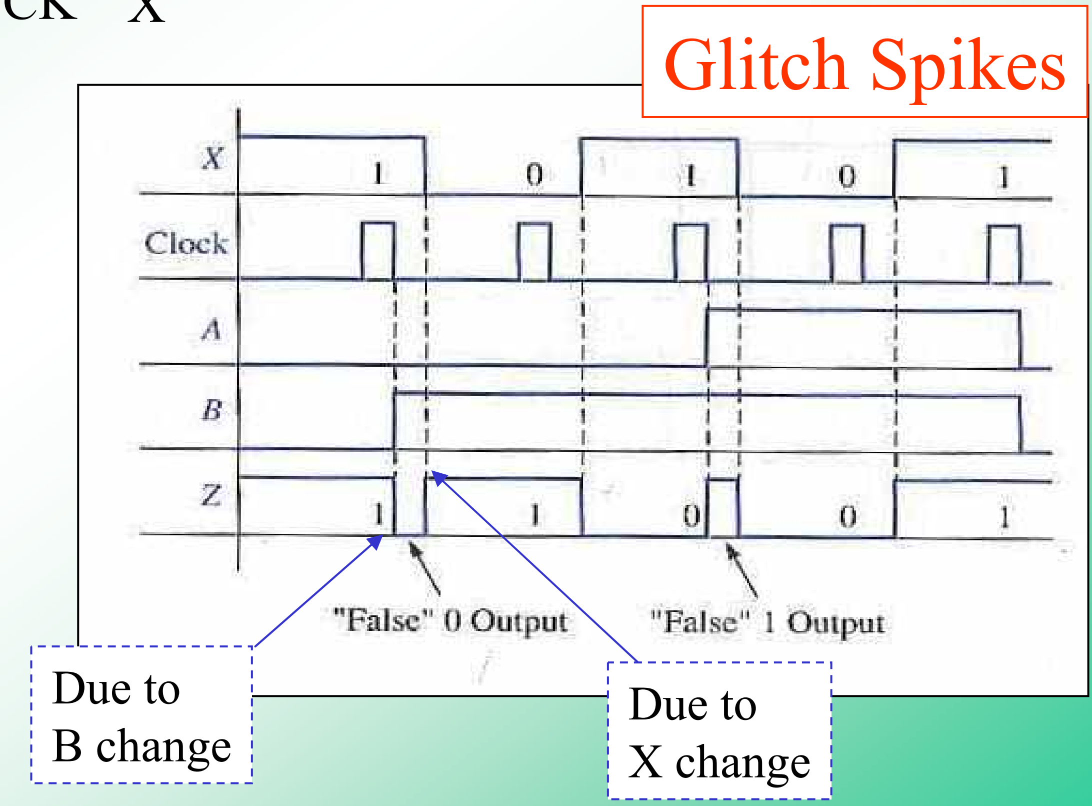


$$X = 10101$$

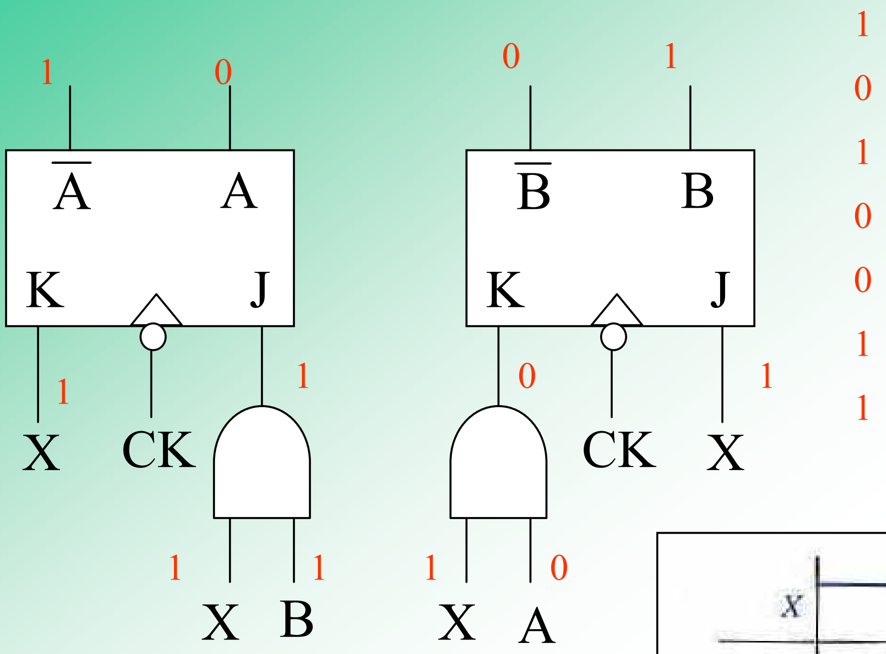
$$A = 000110$$

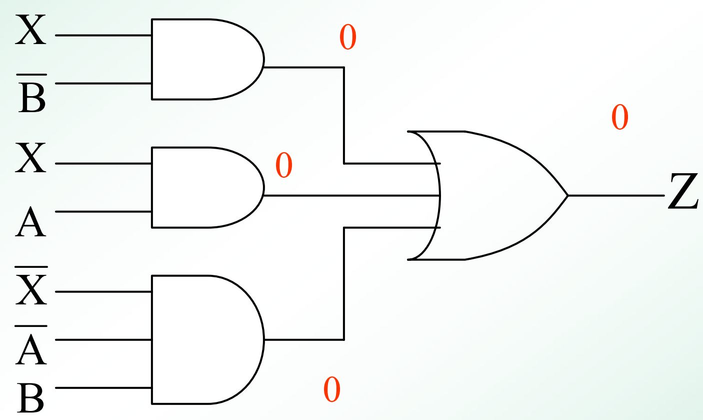
$$B = 011110$$

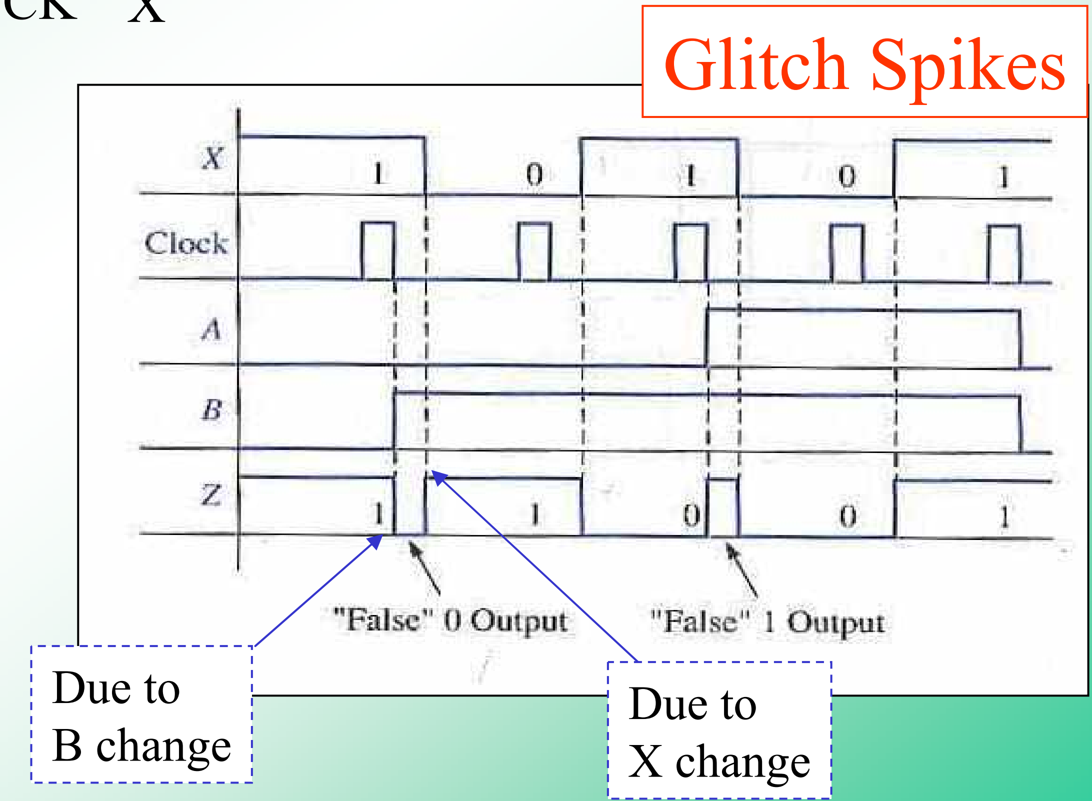
$$Z = 1\,(0)\,10\,(1)\,01$$



Analysis of Mealy Machine







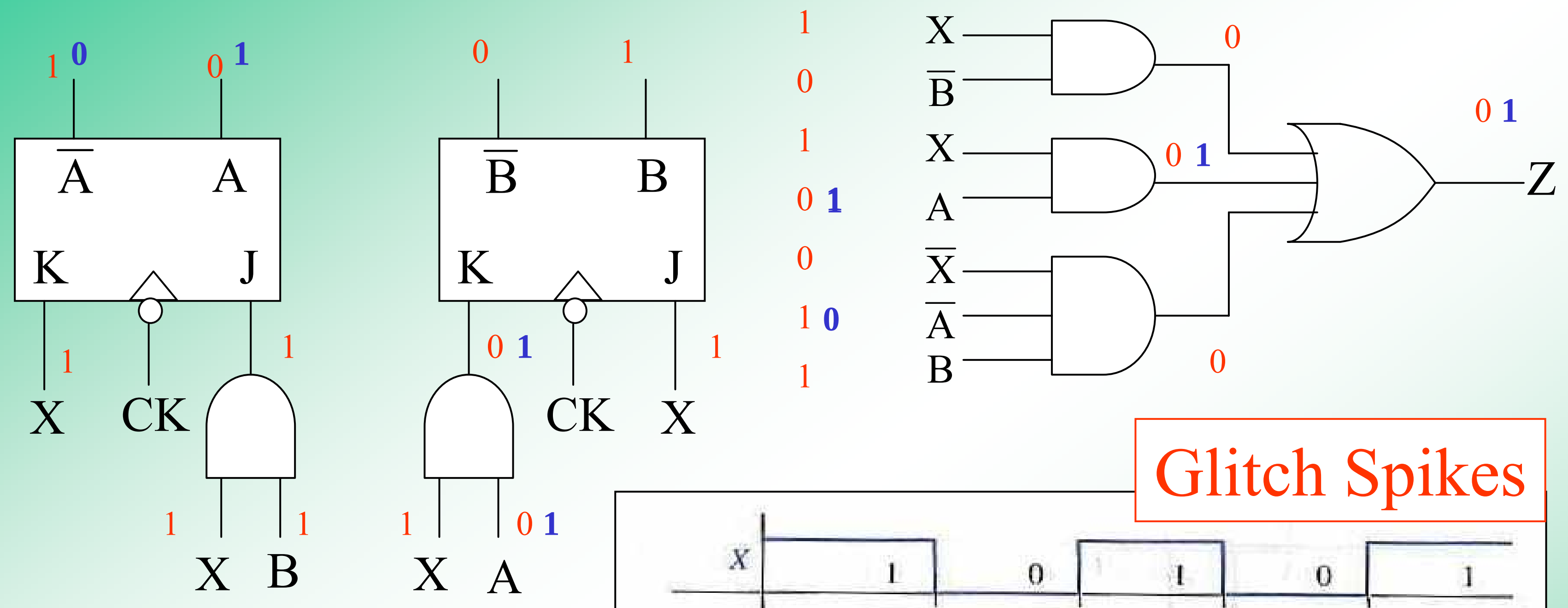
$X = 1\,0\,1\,0\,1$

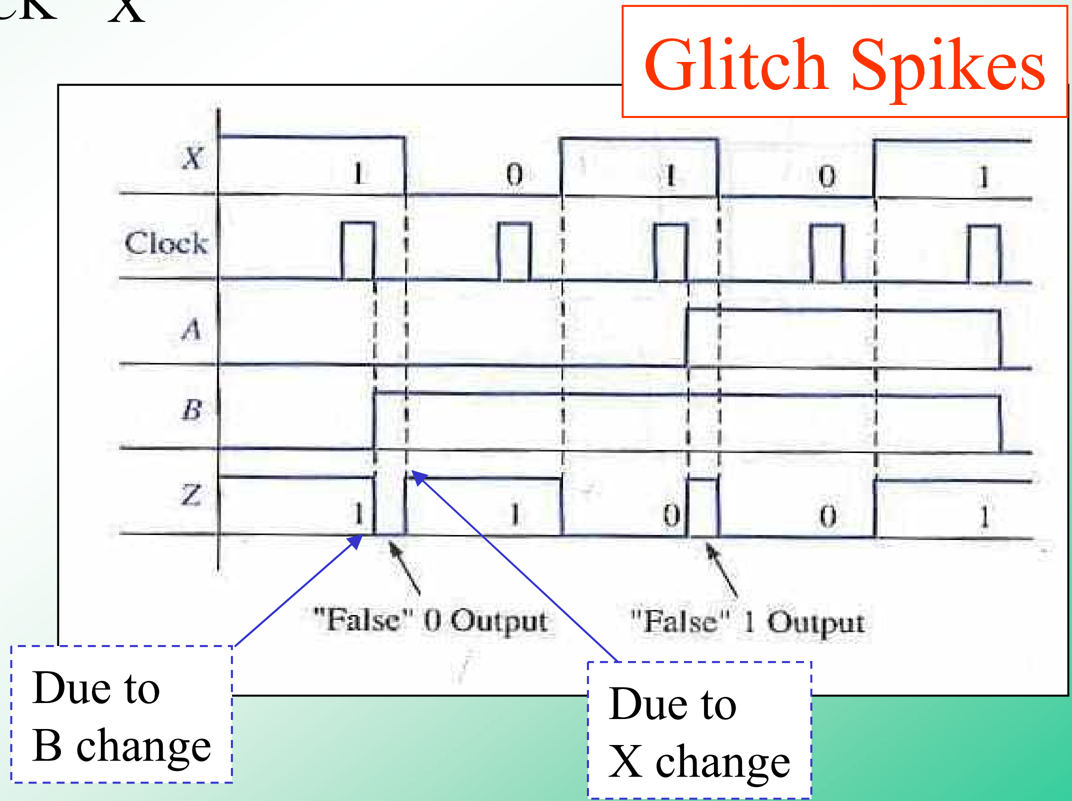
$A = 0\,0\,0\,1\,1\,0$

$B = 0\,1\,1\,1\,1\,0$

$Z = 1\,(0)\,1\,0\,(1)\,0\,1$

Analysis of Mealy Machine





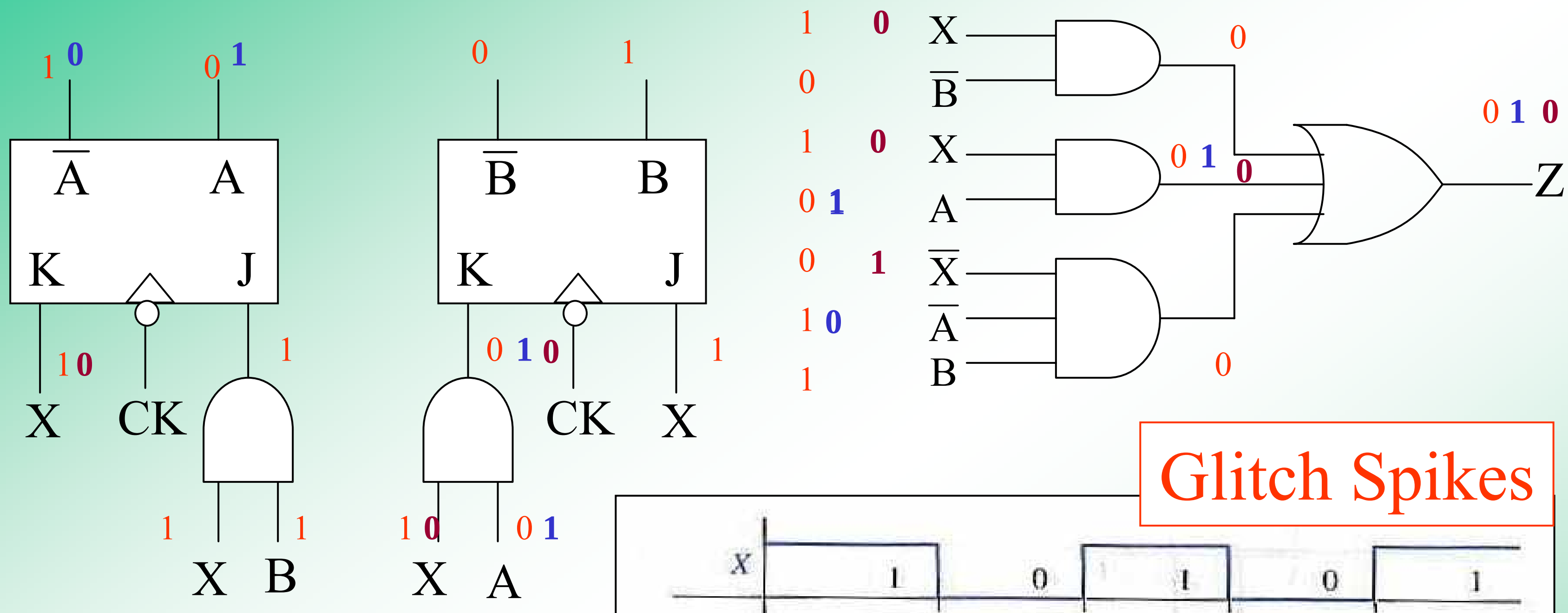
$X = 1\,0\,1\,0\,1$

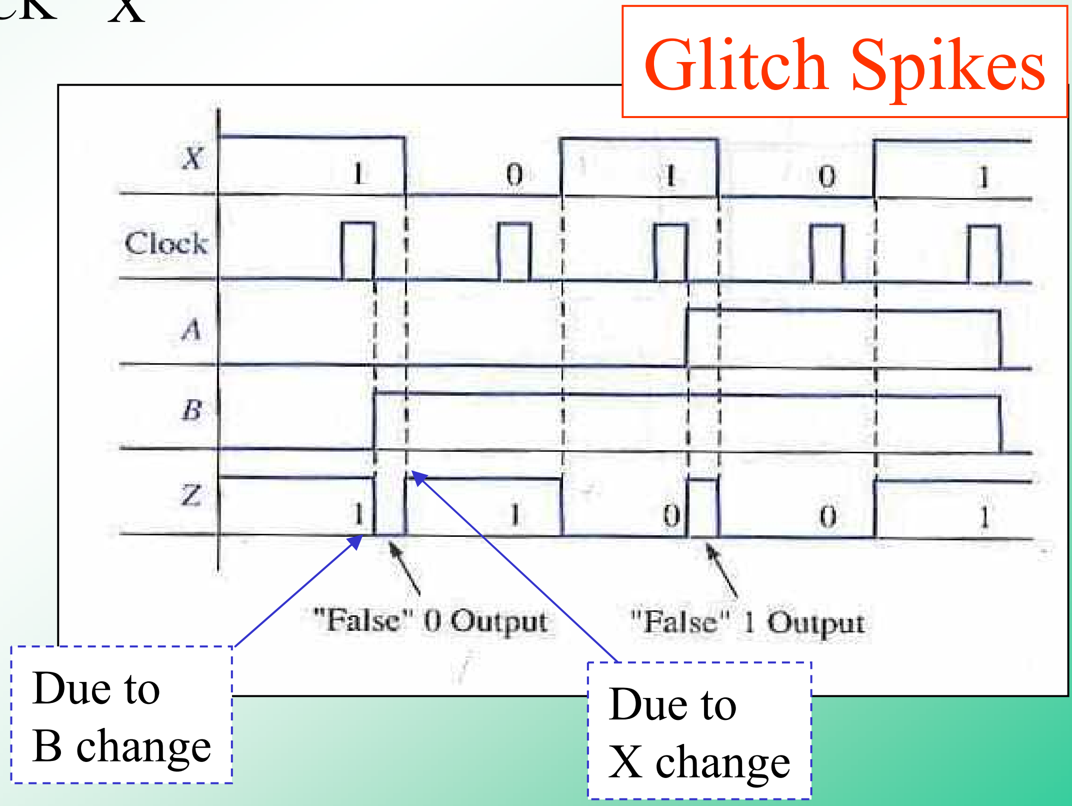
$A = 0\,0\,0\,1\,1\,0$

$B = 0\,1\,1\,1\,1\,0$

$Z = 1\,(0)\,1\,0\,(1)\,0\,1$

Analysis of Mealy Machine





$X = 10101$

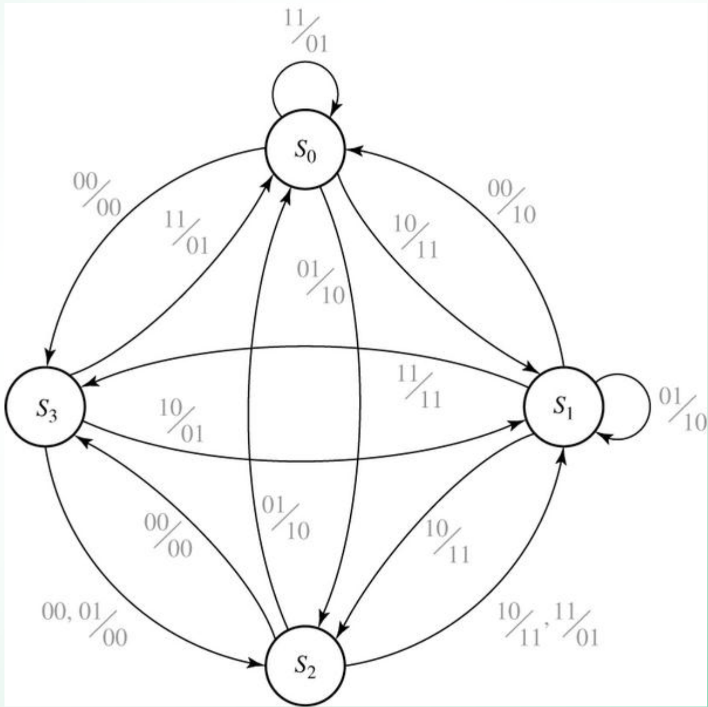
$A = 000110$

$B = 011110$

$Z = 1(0)10(1)01$

A two - input , two - output example

Present state	Next state x_1x_2 = 00	01	10	11	Present output (Z_1Z_2) x_1x_2 = 00	01	10	11
S_0	S_3	S_2	S_1	S_0	00	10	11	01
S_1	S_0	S_1	S_2	S_3	10	10	11	11
S_2	S_3	S_0	S_1	S_1	00	10	11	01
S_3	S_2	S_2	S_1	S_0	00	00	01	01



Construction and Interpretation of Timing Charts

1. When constructing timing charts, note that a state change can only occur after the rising (or falling) edge of the clock.

2. The input will normally be stable immediately before and after the active clock edge.

3. For a **Moore circuit**, the output can change only when the state changes, but for a **Mealy circuit**, the output can change when the input changes as well as when the state changes.

4. False outputs are difficult to determine from the state graph, so use *either* signal tracing through the circuit *or* use the state table when constructing timing charts for **Mealy circuits**.

Construction and Interpretation of Timing Charts

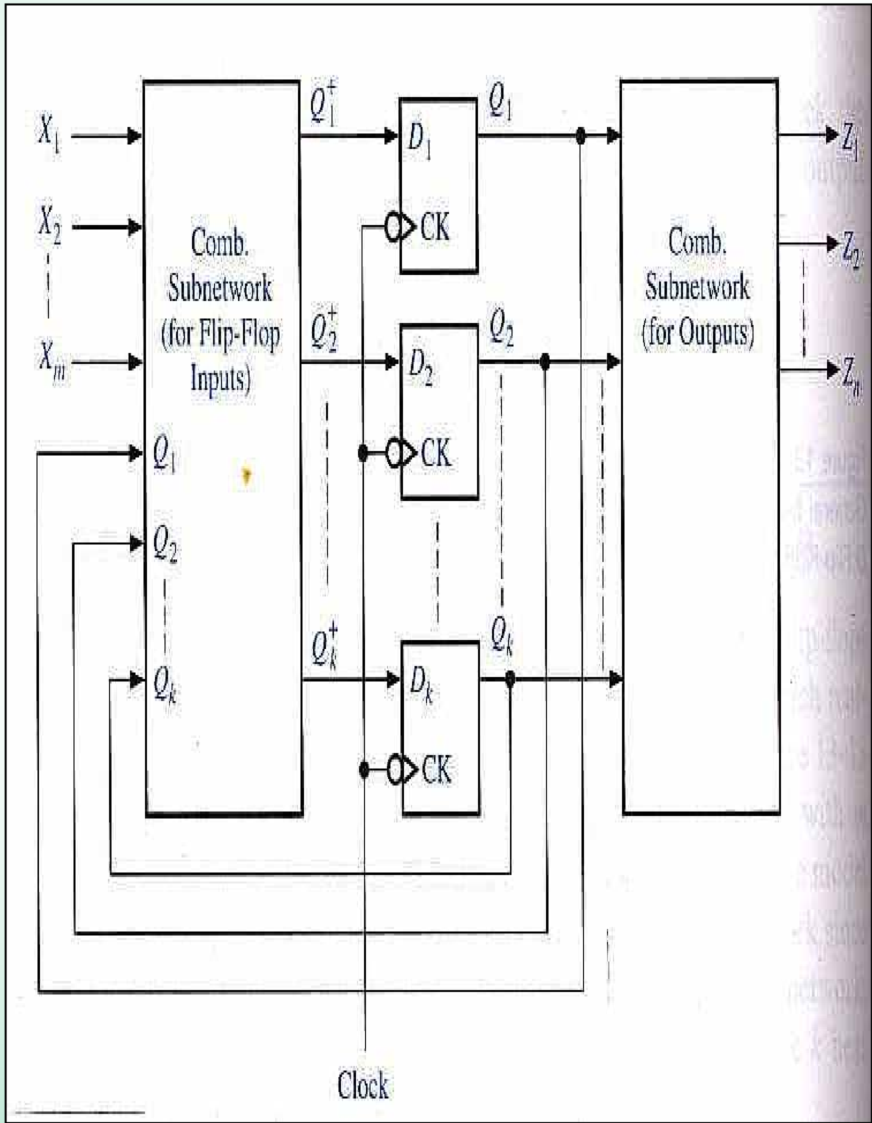
5. When using a Mealy state table for constructing,
   (a) For the first input, read the present output and plot it.
   (b) Read the next state and plot it (following the active edge of the clock)
   (c) Go to the row in the table which corresponds to the next state and read the output under the old input column and plot it. (false output??)
   (d) Change to the next input and repeat steps (a), (b), and (c)

6. For Mealy circuits, the best time to read the output is just before the active edge of the clock, because the output should always be correct at that time.

13-4 General Models for Sequential Circuits

For a Moore machine

$$Z_1 = f_1(Q_1 \cdots Q_k)$$
$$\vdots$$
$$Z_n = f_n(Q_1 \cdots Q_k)$$
$$Q_1^+ = g_1(X_1 \cdots X_m, Q_1 \cdots Q_k)$$
$$\vdots$$
$$Q_k^+ = g_k(X_1 \cdots X_m, Q_1 \cdots Q_k)$$

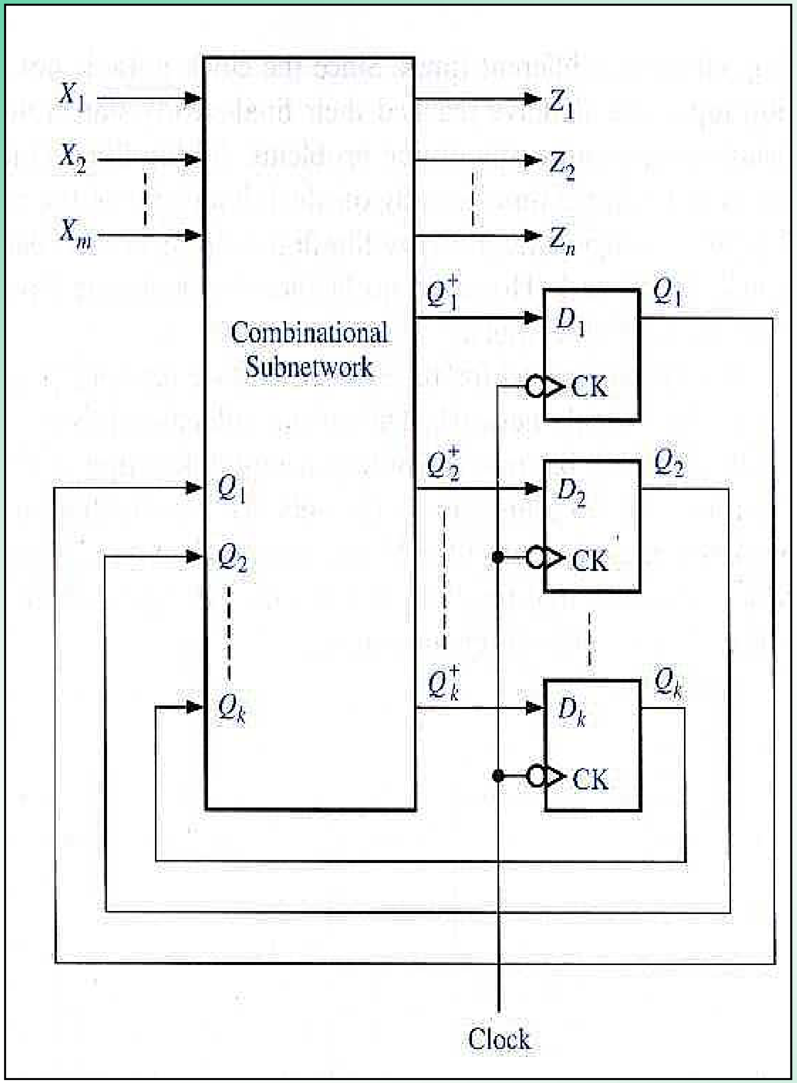


1.當$Q_1^+ \cdots Q_k^+$ settled as clock goes rising or falling edge。

2.當$Q_1 \cdots Q_k$穩定後, $Z_1 \cdots Z_n$即穩定

$\Rightarrow$無Spurious output

For a Mealy machine：D F/Fs are used



$$Z_1 = f_1(X_1 \cdots X_m, Q_1 \cdots Q_k)$$

$$\vdots$$

$$Z_n = f_n(X_1 \cdots X_m, Q_1 \cdots Q_k)$$

$$Q_1^+ = D_1 = g_1(X_1 \cdots X_m, Q_1 \cdots Q_k)$$

$$\vdots$$

$$Q_k^+ = D_k = g_k(X_1 \cdots X_m, Q_1 \cdots Q_k)$$

當$X_1 \cdots X_m$改變，$Q_1 \cdots Q_K$還在Previous state或當$Q_1 \cdots Q_K$已改變，但$X_1 \cdots X_m$ change to new inputs時，可能有 false output 出現。

Present state	Next state X = 0	1	2	3	Present output (Z_1Z_2) X = 0	1	2	3
S_0	S_3	S_2	S_1	S_0	0	2	3	1
S_1	S_0	S_1	S_2	S_3	2	2	3	3
S_2	S_3	S_0	S_1	S_1	0	2	3	1
S_3	S_2	S_2	S_1	S_0	0	0	1	1

← coding of outputs

A state table specifies two functions

- next state function $S^+ = \delta(S, X)$
- present output function $Z = \lambda(S, X)$

From the above table

$\delta(S_0, 1) = S_2$ $\delta(S_0, 3) = S_0$

$\lambda(S_0, 1) = 2$ $\lambda(S_2, 3) = 1$

∵ For J-K F/F : J_A, K_A independent of Q_A 又 $Q^+ = JQ' + K'Q$

$\Rightarrow$ $Q = 0$時 $Q^+ = J$ $\qquad$ $Q = 1$時 $Q^+ = K'$

Next state map : Q_A^+ Q_B^+ Q_C^+

Q_BQ_C \ Q_A	0	1
00	1	1
01	X	X
11	0	0
10	0	X

J_A map | K'_A map

$J_A = B'$ $\quad K_A = B$ or C

Q_BQ_C \ Q_A	0	1
00	1	1
01	X	X
11	0	0
10	0	X

J_B map

B=0

B=1

K'_B map

$J_B=1$ $\quad K_B=1$

Q_BQ_C \ Q_A	0	1
00	1	1
01	X	X
11	0	0
10	0	X

C=0

J_C map

C=1

K'_C map

$J_C=B'$ $\quad K_C=1$

BC \ A	0	1
00	1	1
01	1	X
11	0	0
10	0	0

A^+

BC \ A	0	1
00	0	1
01	1	1
11	0	1
10	1	X

B^+

BC \ A	0	1
00	1	1
01	1	X
11	0	0
10	0	0

C^+

??????

BC \ A	0	1
00	1	
01	1	
11		1
10		1

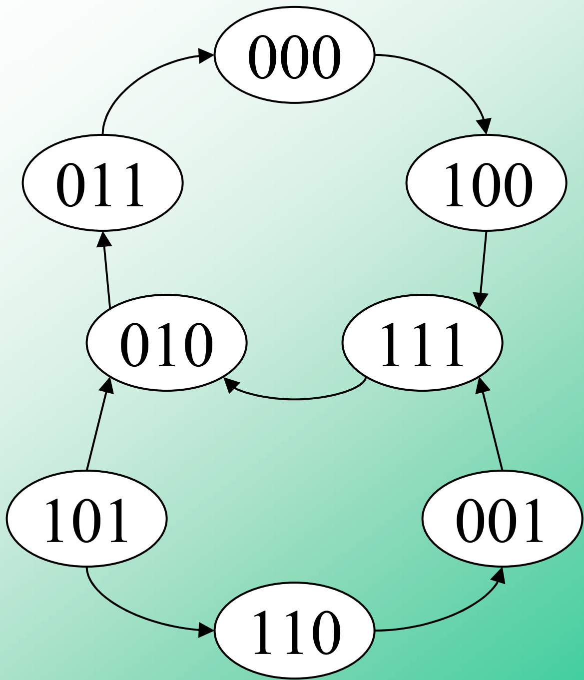
T_A

BC \ A	0	1
00		1
01	1	1
11	1	
10	1	

T_B

BC \ A	0	1
00		1
01		1
11	1	1
10	1	1

T_C



HOMEWORK -- Unit 13

- 13.3
- 13.7
- 13.13
- 13.17