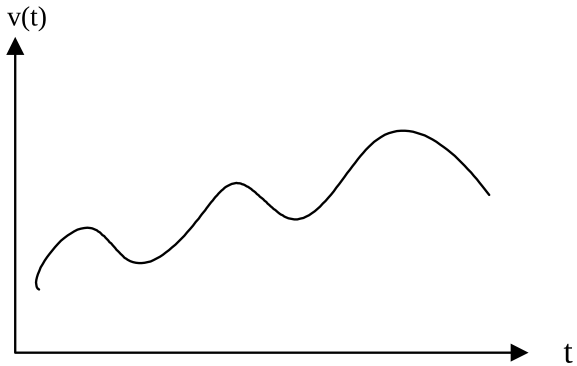
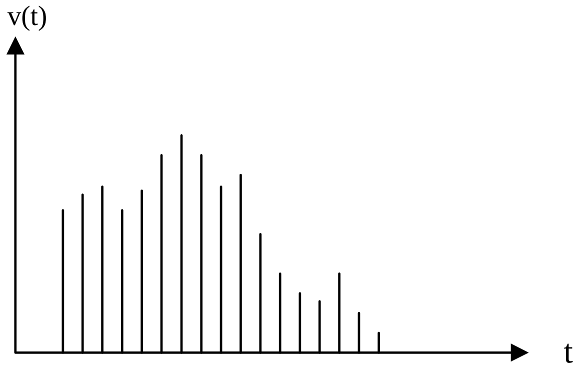
Unit 1. Number Systems and Conversion

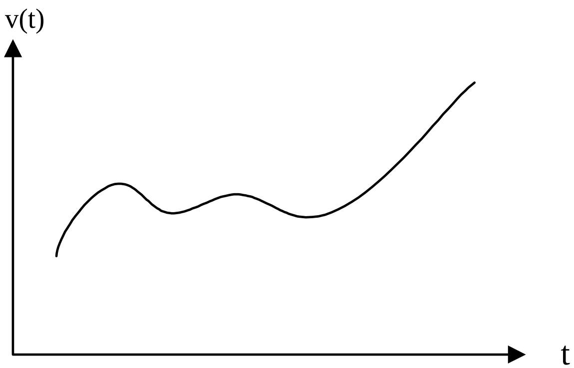
§ **Digital Systems and Switching Circuits**



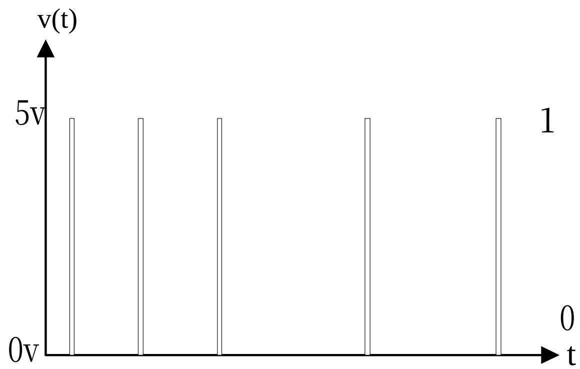
Continuous signal



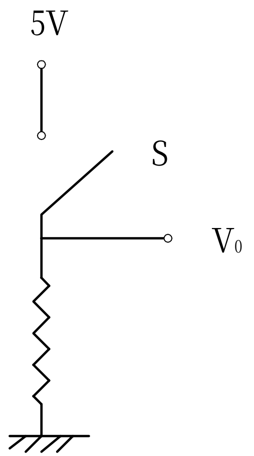
Discrete signal



Analog signal



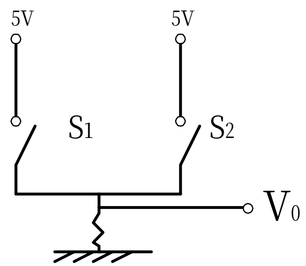
Digital signal



S ON “1”(5V)

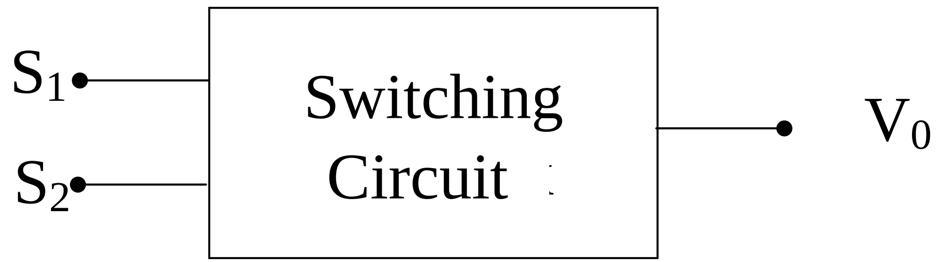
OFF “0”(0V)

Switching circuit



S_1 OR S_2 ON (5V) “1”

S_1 AND S_2 OFF (0V) “0”



- Combinational Circuit
  - V_0 ： function of S_1, S_2 present values

- Sequential Circuit
  - V_0 ： function of S_1, S_2 both present & previous values
  - memory behavior

- Switches
  - realized by transistors
  - transistor level, gate level, module level (adder, arithmetic unit….)

§ Number Systems & Conversion

- Number Systems
  - Base 10：$953.78_{10} = 9\times10^{2}+5\times10^{1}+3\times10^{0}+7\times10^{-1}+8\times10^{-2}$
    (Decimal)
  - Base 2：$1011.11_{2} = 1\times2^{3}+0\times2^{2}+1\times2^{1}+1\times2^{0}+1\times2^{-1}+1\times2^{-2}$
    (Binary)
    $= 8+0+2+1+\frac{1}{2}+\frac{1}{4} = 11\frac{3}{4} = 11.75_{10}$

- Base R : (0,1,⋯,R-1)   Base 4 : (0,1,2,3)
- $$N = (a_4 a_3 a_2 a_1 a_0 . a_{-1} a_{-2} a_{-3})_R$$
  $$= a_4 * R^4 + a_3 * R^3 + a_2 * R^2 + a_1 * R^1 + a_0 * R^0$$
  $$+ a_{-1} * R^{-1} + a_{-2} * R^{-2} + a_{-3} * R^{-3} \quad , \quad a_i < R$$
- $$147.3_8 = 1 \times 8^2 + 4 \times 8^1 + 7 \times 8^0 + 3 \times 8^{-1} = 64 + 32 + 7 + \frac{3}{8}$$
  $$= 103.375_{10}$$ Octal
- Base16:(0,1,2,…,9,A,B,C,D,E,F)   Hexadecimal
- $$\text{A2F}_{16} = 10 \times 16^2 + 2 \times 16^1 + 15 \times 16^0 = 2560 + 32 + 15$$
  $$= 2607_{10}$$

- Conversion

  Decimal $\Rightarrow$ R

  53_{10} $\Rightarrow$ Binary

$a_5\ a_4\ a_3\ a_2\ a_1\ a_0$

$53_{10} = 1\ 1\ 0\ 1\ 0\ 1_2$

2)	53…….		
2)	26 …….	1	a_0
2)	13 …….	0	a_1
2)	6 …….	1	a_2
2)	3 …….	0	a_3
2)	1 …….	1	a_4
	0 …….	1	a_5

- $N_{10} = (a_n a_{n-1} \ldots a_2 a_1 a_0)_R = a_n R^n + a_{n-1} R^{n-1} + \ldots\ldots + a_2 R^2 + a_1 R^1 + a_0$

- $\dfrac{N_{10}}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \ldots\ldots + a_2 R + a_1$ -------------- *Remainder* a_0

- $\dfrac{N_{10}}{R^2} = a_n R^{n-2} + \ldots\ldots\ a_3 R + a_2$ -------------------------- *Remainder* a_1

- $\dfrac{N_{10}}{R^3} = a_n R^{n-3} + \ldots\ldots + a_3$ ---------------------------------- *Remainder* a_2

$\vdots$

$\vdots$

$$F_{10} = (.a_{-1}\ a_{-2} \ldots.a_{-m})_R = a_{-1} R^{-1} + a_{-2}R^{-2} + \ldots\ldots + a_{-m} R^{-m}$$

$$FR = a_{-1} + a_{-2}R^{-1} + \ldots\ldots + a_{-m} R^{-m+1} = a_{-1} + F_1$$

$$F_1R = a_{-2} + a_{-3}R^{-1} + \ldots\ldots + a_{-m} R^{-m+2} = a_{-2} + F_2$$

$$F_2R = a_{-3} + F_3$$

⋮

⋮

$$Ex: .625_{10} \Rightarrow .(\qquad)_2$$

$$F = .625 \qquad F_1 = .250 \qquad F_2 = .50$$

$$\times\ 2 \qquad \times\ 2 \qquad \times\ 2$$

$$1.250 \qquad 0.500 \qquad 1.000$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$a_{-1} \qquad a_{-2} \qquad a_{-3}$$

$$\Rightarrow .101_2$$

$$0.7_{10} = (\,0.10110\ldots\ldots)_2$$

$$
\begin{array}{r}
.7 \\
\times\ 2 \\
\hline
(1).4 \\
\times\ 2 \\
\hline
(0).8 \\
\times\ 2 \\
\hline
1.6 \\
\times\ 2 \\
\hline
1.2 \\
\times\ 2 \\
\hline
0.4
\end{array}
$$

a_{-1} → (1).4

a_{-2} → (0).8

a_{-3} → 1.6

a_{-4} → 1.2

a_{-5} → 0.4

$$Ex. \quad 231.3_4 \Rightarrow (\ 63.5151\ )_7$$

$$231.3_4 = 2\times 4^2 + 3\times 4^1 + 1\times 4^0 + 3\times \tfrac{1}{4} = 45.75_{10}$$

$$\begin{array}{r|rr l}
7\,) & 4 & 5 & \\ \hline
7\,) & & 6 & \cdots\cdots 3 \\ \hline
 & & 0 & \cdots\cdots 6
\end{array}$$

$$\begin{array}{llll}
 & . & 7 & 5 \\
 & & & 7 \\ \hline
(5) & . & 2 & 5 \\
 & & & 7 \\ \hline
(1) & . & 7 & 5 \\
 & & & 7 \\ \hline
(5) & . & 2 & 5 \\
 & & & 7 \\ \hline
(1) & . & 7 & 5
\end{array}$$

$$45.75_{10} = (63.5151)_7$$

Binary to Octal

$$\begin{array}{cccccccccc} 1 & \underleftrightarrow{101} & \underleftrightarrow{011} & \underleftrightarrow{101} & \underleftrightarrow{110} & . & \underleftrightarrow{001} & \underleftrightarrow{100}_{2} & = 15356.14_8 \\ 1 & 5 & 3 & 5 & 6 & . & 1 & 4_{8} & \end{array}$$

Binary to Hexadecimal

$$\begin{array}{ccccccc} 1 & \underleftrightarrow{1010} & \underleftrightarrow{1110} & \underleftrightarrow{1110} & . & \underleftrightarrow{0011}_2 & = 1AEE.3_{16} \\ 1 & A & E & E & . & 3 & \end{array}$$

Binary Arithmetic

Addition

				1	1	1		
	13_{10}	=		1	1	0	1	
+	11_{10}	=		1	0	1	1	
			1	1	0	0	0	$=24_{10}$

Subtraction

	1	1	1	0	1
–	1	0	0	1	1
	0	1	0	1	0

				1	1	0	1	=	13_{10}	
*				1	0	1	1	=	11_{10}	
				1	1	0	1			
			1	1	0	1				
		0	0	0	0					
	1	1	0	1						
1	0	0	0	1	1	1	1	=	143_{10}	*Multiplica tion*

Division						1	1	0	1
1011)	1	0	0	0	1	1	1	1	
		1	0	1	1				
		0	1	1	0	1	1	1	
			1	0	1	1			
				0	1	0	1	1	
					1	0	1	1	
								0	

Negative Numbers

+N	Positive integers (all systems)	-N	Negative integers		
			Sign and magnitude	2's complement	1's complement
+0 +1 +2 +3 +4 +5 +6 +7	0000 0001 0010 0011 0100 0101 0110 0111	-0 -1 -2 -3 -4 -5 -6 -7 -8	1000 1001 1010 1011 1100 1101 1110 1111 -----	----- 1111 1110 1101 1100 1011 1010 1001 1000	1111 1110 1101 1100 1011 1010 1001 1000 -----

- Sign-magnitude

  (sign bit) + (postive magnitude)

  Hard to implement in hardware
- 1's complement

  $\overline{N} = (2^n - 1) - N$
- 2's complement

  $N^* = 2^n - N$, For n = 4, -N = 16-N

  e.g. -3 => 16-3 = 13 = 1101_2
- Easy way to form 1's complement
  - Complementing N bit-by-bit
- Easy way to form 2's complement
  - Complementing N bit-by-bit and then adding 1

Binary Codes

9	3	7	2	5
1001	0011	0111	0010	0101

Binary-Coded-Decimal(BCD)
8-4-2-1 BCD

Why Binary Codes

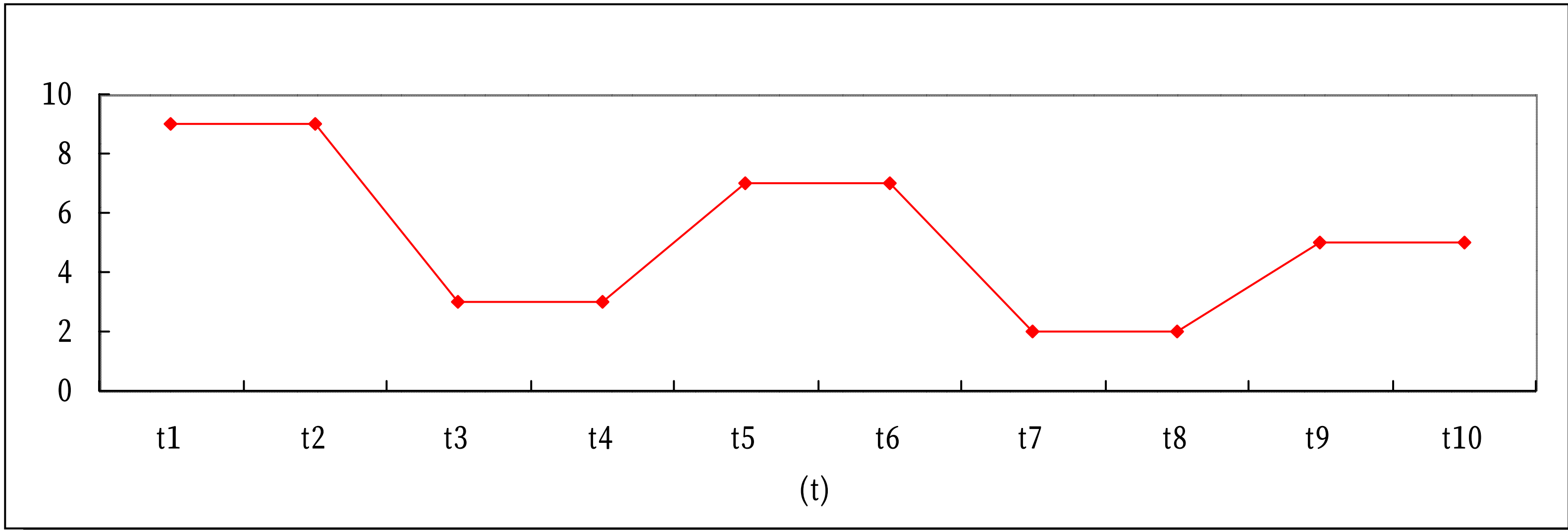


Table 1-1 Various Codes

Decimal Digit	$8-4-2-1$ Code (BCD)	$6-3-1-1$ Code	Excess-3 Code	2-out-of-5 Code	Gray Code
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	1110
6	0110	1000	1001	10001	1010
7	0111	1001	1010	10010	1011
8	1000	1011	1011	10100	1001
9	1001	1100	1100	11000	1000

Weighted Codes

$$8 \quad 4 \quad 2 \quad 1$$

$$6 \quad 3 \quad 1 \quad 1 \qquad Ex.\ 1011 = 6+1+1=8$$

Excess-3 Codes

$$8 \quad 4 \quad 2 \quad 1 \quad + \quad 3(0011)\ \mathit{to\ each\ codes}$$

$$0 \quad 0 \quad 1 \quad 1 \rightarrow 0110$$

2-out-of-5 Codes

2 out of 5 digits are “1”

Error-correcting Codes

Gray Codes

Each Bit differ by 1-bit

Table 1–2
ASCII Code

Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0	Character	A_6	A_5	A_4	A_3	A_2	A_1	A_0
space	0	1	0	0	0	0	0	@	1	0	0	0	0	0	0	‘	1	1	0	0	0	0	0
!	0	1	0	0	0	0	1	A	1	0	0	0	0	0	1	a	1	1	0	0	0	0	1
″	0	1	0	0	0	1	0	B	1	0	0	0	0	1	0	b	1	1	0	0	0	1	0
#	0	1	0	0	0	1	1	C	1	0	0	0	0	1	1	c	1	1	0	0	0	1	1
$	0	1	0	0	1	0	0	D	1	0	0	0	1	0	0	d	1	1	0	0	1	0	0
%	0	1	0	0	1	0	1	E	1	0	0	0	1	0	1	e	1	1	0	0	1	0	1
&	0	1	0	0	1	1	0	F	1	0	0	0	1	1	0	f	1	1	0	0	1	1	0
’	0	1	0	0	1	1	1	G	1	0	0	0	1	1	1	g	1	1	0	0	1	1	1
(	0	1	0	1	0	0	0	H	1	0	0	1	0	0	0	h	1	1	0	1	0	0	0
)	0	1	0	1	0	0	1	I	1	0	0	1	0	0	1	i	1	1	0	1	0	0	1
*	0	1	0	1	0	1	0	J	1	0	0	1	0	1	0	j	1	1	0	1	0	1	0
+	0	1	0	1	0	1	1	K	1	0	0	1	0	1	1	k	1	1	0	1	0	1	1
,	0	1	0	1	1	0	0	L	1	0	0	1	1	0	0	l	1	1	0	1	1	0	0
−	0	1	0	1	1	0	1	M	1	0	0	1	1	0	1	m	1	1	0	1	1	0	1
.	0	1	0	1	1	1	0	N	1	0	0	1	1	1	0	n	1	1	0	1	1	1	0
/	0	1	0	1	1	1	1	O	1	0	0	1	1	1	1	o	1	1	0	1	1	1	1
0	0	1	1	0	0	0	0	P	1	0	1	0	0	0	0	p	1	1	1	0	0	0	0
1	0	1	1	0	0	0	1	Q	1	0	1	0	0	0	1	q	1	1	1	0	0	0	1
2	0	1	1	0	0	1	0	R	1	0	1	0	0	1	0	r	1	1	1	0	0	1	0
3	0	1	1	0	0	1	1	S	1	0	1	0	0	1	1	s	1	1	1	0	0	1	1
4	0	1	1	0	1	0	0	T	1	0	1	0	1	0	0	t	1	1	1	0	1	0	0
5	0	1	1	0	1	0	1	U	1	0	1	0	1	0	1	u	1	1	1	0	1	0	1
6	0	1	1	0	1	1	0	V	1	0	1	0	1	1	0	v	1	1	1	0	1	1	0
7	0	1	1	0	1	1	1	W	1	0	1	0	1	1	1	w	1	1	1	0	1	1	1
8	0	1	1	1	0	0	0	X	1	0	1	1	0	0	0	x	1	1	1	1	0	0	0
9	0	1	1	1	0	0	1	Y	1	0	1	1	0	0	1	y	1	1	1	1	0	0	1
:	0	1	1	1	0	1	0	Z	1	0	1	1	0	1	0	z	1	1	1	1	0	1	0
;	0	1	1	1	0	1	1	[	1	0	1	1	0	1	1	{	1	1	1	1	0	1	1
<	0	1	1	1	1	0	0	\	1	0	1	1	1	0	0	|	1	1	1	1	1	0	0
=	0	1	1	1	1	0	1	]	1	0	1	1	1	0	1	}	1	1	1	1	1	0	1
>	0	1	1	1	1	1	0	^	1	0	1	1	1	1	0	~	1	1	1	1	1	1	0
?	0	1	1	1	1	1	1	_	1	0	1	1	1	1	1	delete	1	1	1	1	1	1	1

Homework

1.1 1.3 1.5 1.7 1.9 1.11 1.15 1.18 1.20 1.22

1.25 1.28