

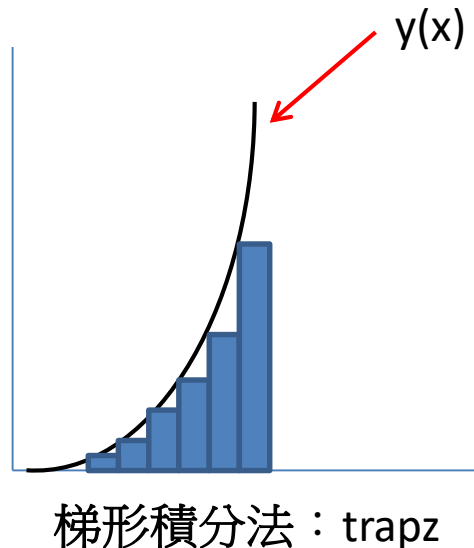
# Matlab

## 微積分應用

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# 應用1: 求函數積分

- 不知函數 $f(x)$ ，已知數據點：
  - trapz: 梯形積分法
- 已知函數 $f(x)$ ：
  - quad: 適應性辛普森法



## 基本積分表

$$\int k dx = kx + C$$

$$\int ax^n dx = a \frac{1}{n+1} x^{n+1} + C \quad (n \neq -1)$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int e^x dx = e^x + C$$

$$\int \frac{k}{x} dx = k \ln |x| + C$$

# 數值積分

- 梯形積分法

**A = trapz(x,y)**

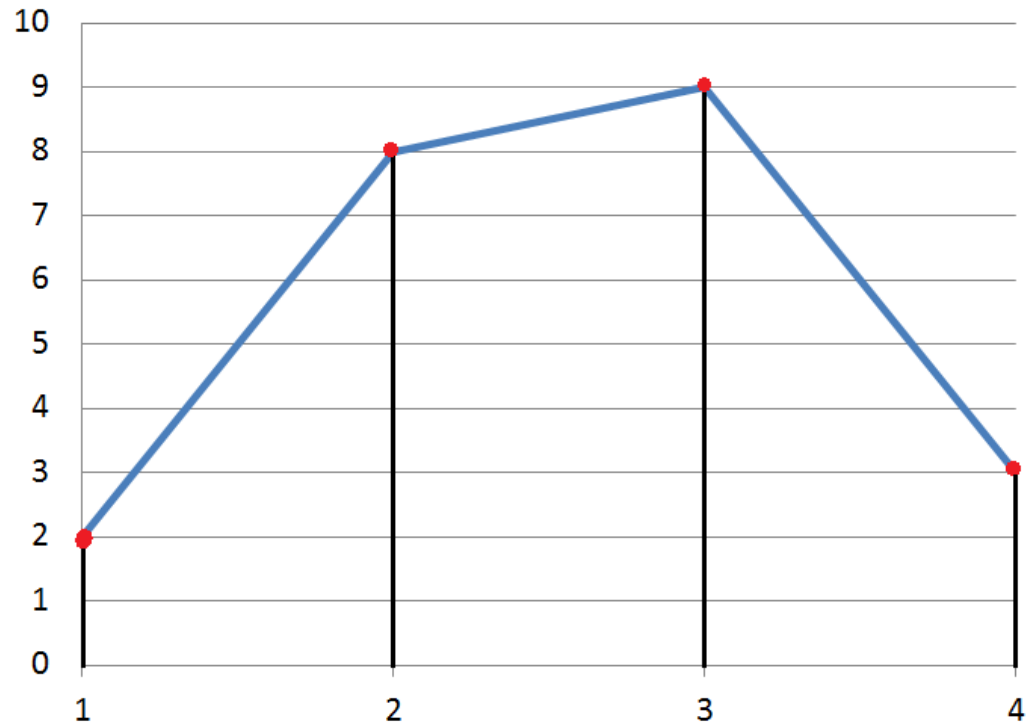
— Ex:

```
>> x = [1 2 3 4];
```

```
>> y = [2 8 9 3];
```

```
>> area = trapz(x,y)
```

```
area = 19.5
```



# 數值積分

- 已知函數積分：quad

**A = quad('func',a,b)**

Ex1:  $\int_0^2 \frac{1}{x^3 - 2x - 5} dx$

Func: 函式  
a: 積分下限  
b: 積分上限

```
>> area = quad('1./(x.^3-2*x-5)',0,2)
area = -0.4605
```

Ex2:  $\int_1^2 \exp(2x) dx$

```
>> A = quad('exp(2*x)',1,2)
A = 23.605
```

**NOTE:**

函數內之數學運算必須使用向量個別元素之運算  
(.\* ./ .^)

# 應用2: 求函數微分

## 基本微分表

$$\frac{d}{dx} kx^n = knx^{n-1}$$

$$\frac{d}{dx} a^x = a^x \ln a$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{d}{dt} (6 \sin (4t)) = 6 \times \frac{d}{dt} (\sin (4t))$$

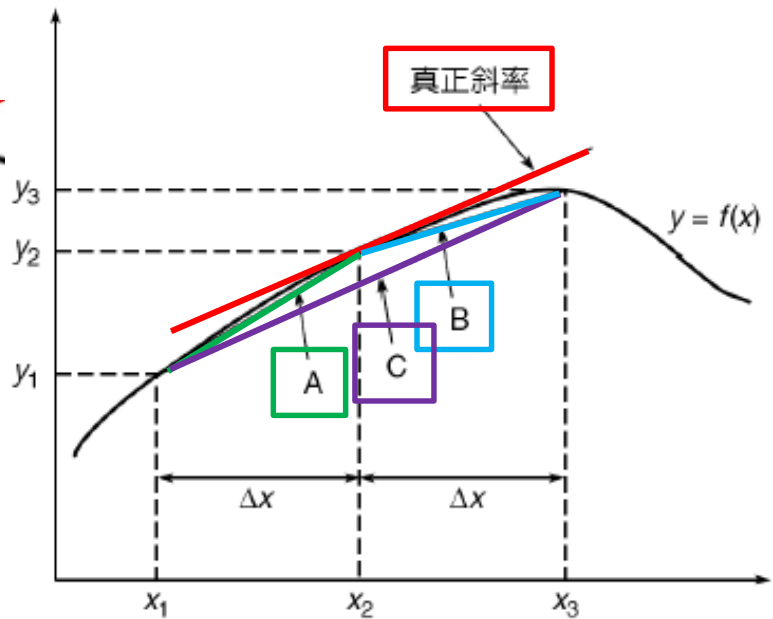
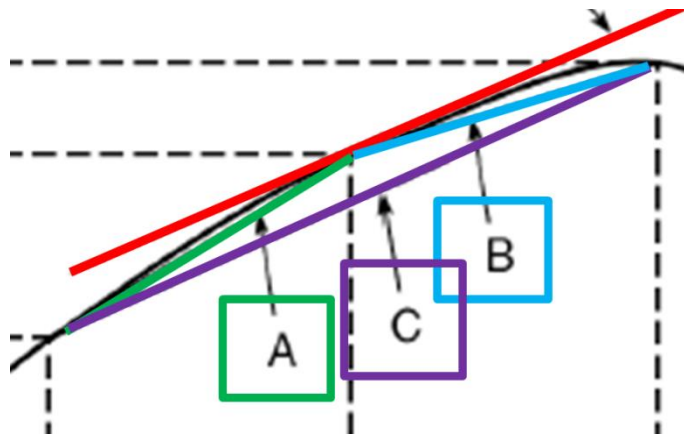
$$= 6 \times \cos(4t) \times \frac{d}{dt} (4t)$$

$$= 6 \times \cos(4t) \times 4$$

$$= 24 \cos(4t)$$

# 數值微分

## ■ 已知數據點的微分



## ■ 在 $x_2$ 之微分

Definition of Derivative:  $\frac{dy}{dx}$

Backward Difference:  $m_A$

Forward Difference:  $m_B$

Central Difference:  $m_C$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{\Delta x}$$

$$= \frac{y_3 - y_2}{x_3 - x_2} = \frac{y_3 - y_2}{\Delta x}$$

$$= \frac{1}{2} \left( \frac{y_3 - y_2}{\Delta x} + \frac{y_2 - y_1}{\Delta x} \right) = \frac{y_3 - y_1}{2\Delta x}$$

Ex:  $6\sin(4t)$ :

```
>> x = linspace(0,pi,40);
```

```
>> y = 6*sin(4*x);
```

```
>> d = diff(y)./diff(x); % backward or forward difference
```

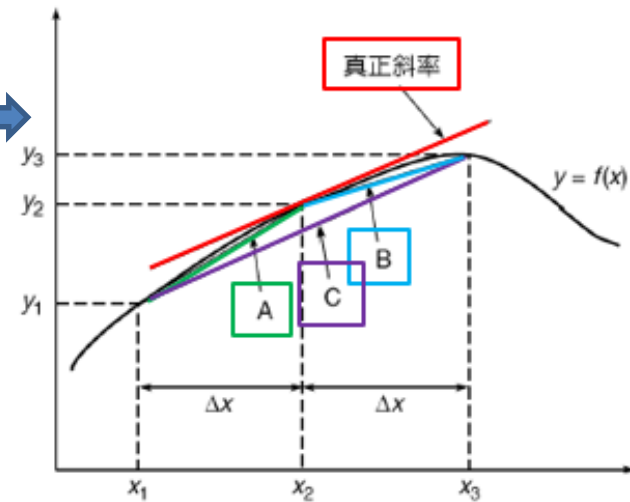
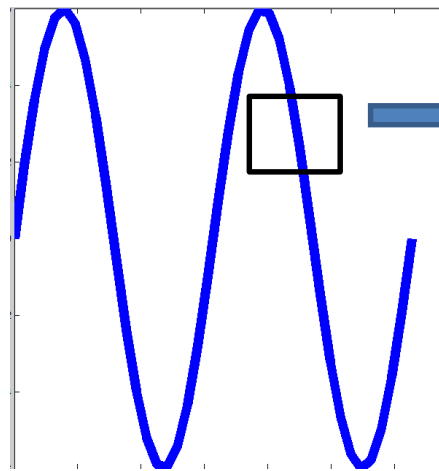
```
>> dc = (y(3:end)-y(1:end-2))./(x(3:end)-x(1:end-2)); % central difference
```

```
>> dy = 24*cos(4*x); % 實際微分值
```

```
>> plot(x, dy, x(2:end), d,'o', x(1:end-1), d,'x', x(2:end-1), dc,'^')
```

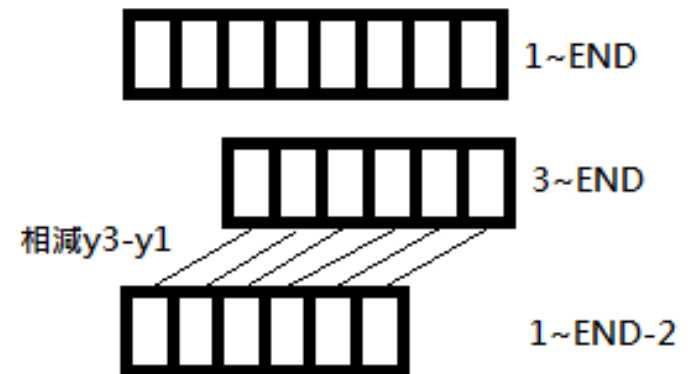
```
>> xlabel('x','FontSize',[60]);
```

```
>> ylabel('Derivative','FontSize',[60])
```

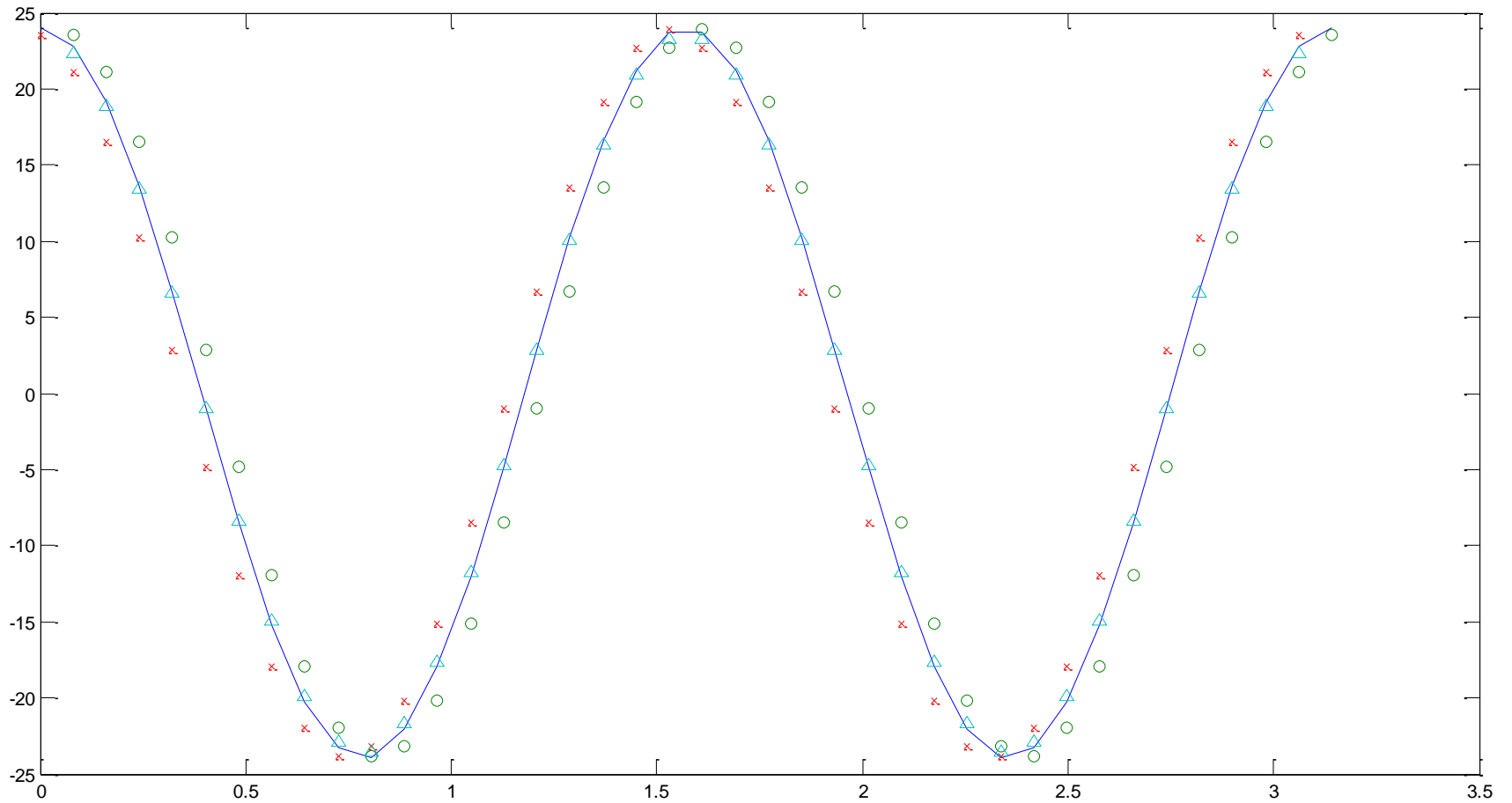


$$d = \text{diff}(x)$$

$$= [x(2) - x(1), x(3) - x(2), \dots, x(n) - x(n-1)]$$



# Derivative



**X**

O = backward difference

X = forward difference

Δ = central difference

-- =  $24 \cdot \cos(4 \cdot x)$