Matlab_3
微積分應用

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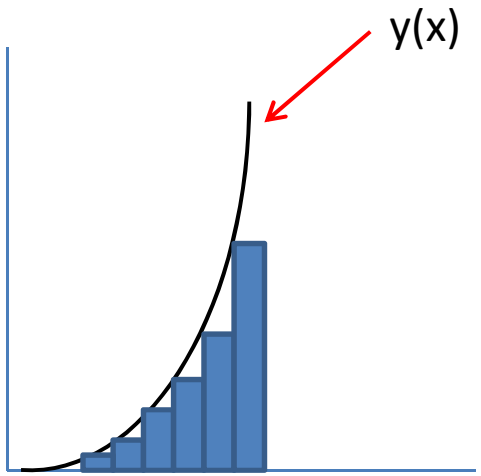
應用3-1: 求函數積分

- 不知函數f(x)，已知數據點：
  - trapz:梯形積分法
- 已知函數f(x)：
  - quad: 適應性辛普森法



梯形積分法：trapz

基本積分表

$$\int k\,dx = kx + C$$

$$\int ax^n dx = a\frac{1}{n+1}x^{n+1} + C \quad (n \neq -1)$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

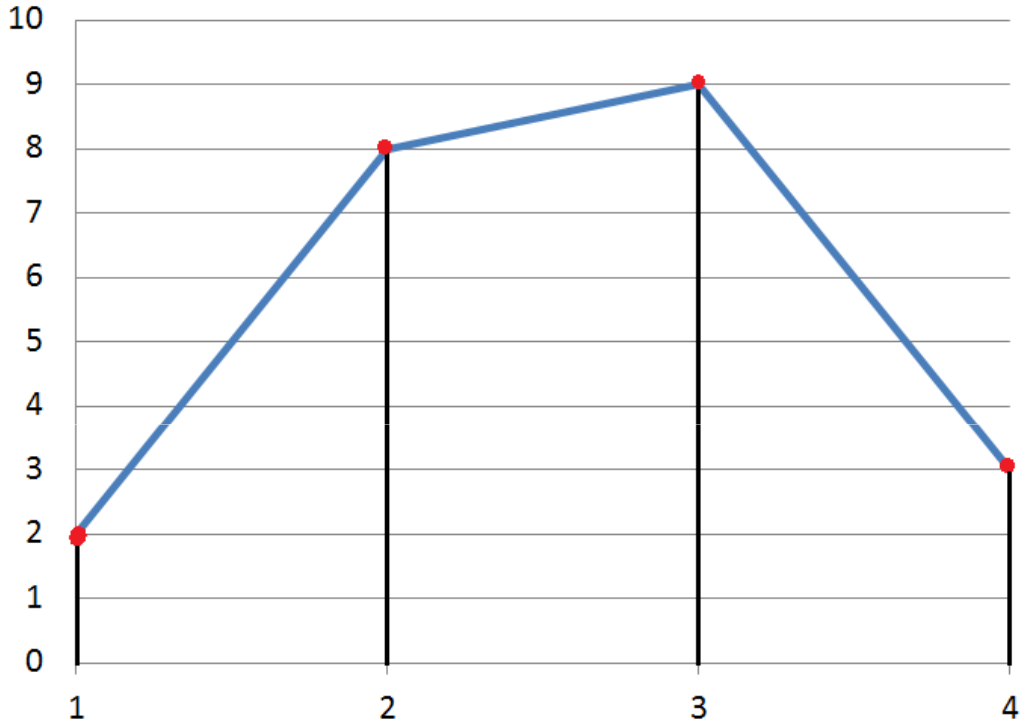
$$\int e^x dx = e^x + C$$

$$\int \frac{k}{x}dx = k\ln|x| + C$$

數值積分

- 梯形積分法

A = trapz(x,y)



– Ex:

```
>>  x = [1 2 3 4];
>>  y = [2 8 9 3];
>>  area = trapz(x,y)
area = 19.5
```

數值積分

- 已知函數積分：quad

A = quad('func',a,b)

Func: 函式
a: 積分下限
b: 積分上限

Ex1: $\int_0^2 \frac{1}{x^3 - 2x - 5} dx$

```
>> area = quad('1./(x.^3-2*x-5)',0,2)
area = -0.4605
```

Ex2: $\int_1^2 \exp(2x)dx$

```
>> A = quad('exp(2*x)',1,2)
A =  23.605
```

NOTE:
函數內之數學運算必須使用向量個別元素之運算
(.* ./ .^)

應用3-2: 求函數微分

基本微分表
$\frac{\mathrm{d}}{\mathrm{d}x}kx^n = knx^{n-1}$
$\frac{\mathrm{d}}{\mathrm{d}x}a^x = a^x ln a$
$\frac{\mathrm{d}}{\mathrm{d}x}e^x = e^x$
$\frac{\mathrm{d}}{\mathrm{d}x}ln\ x = \frac{1}{x}$
$\frac{\mathrm{dy}}{\mathrm{dt}} = \frac{\mathrm{dy}}{\mathrm{dx}} \times \frac{\mathrm{dx}}{\mathrm{dt}}$

$$\frac{\mathrm{d}}{\mathrm{dt}}(6\sin(4t)) = 6 \times \frac{\mathrm{d}}{\mathrm{dt}}(\sin(4t))$$

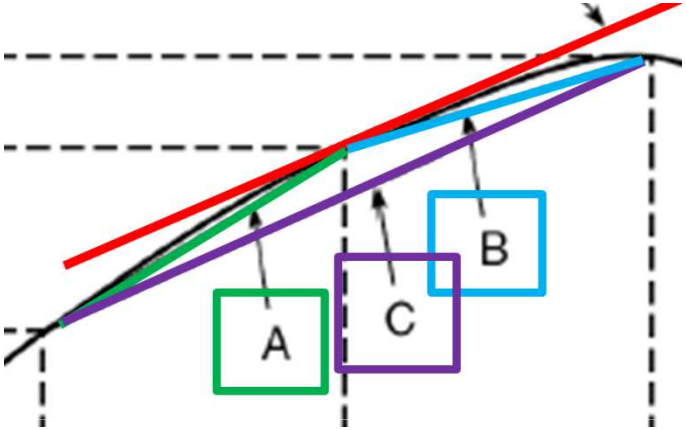
$$= 6 \times \cos(4t) \times \frac{\mathrm{d}}{\mathrm{dt}}(4t)$$

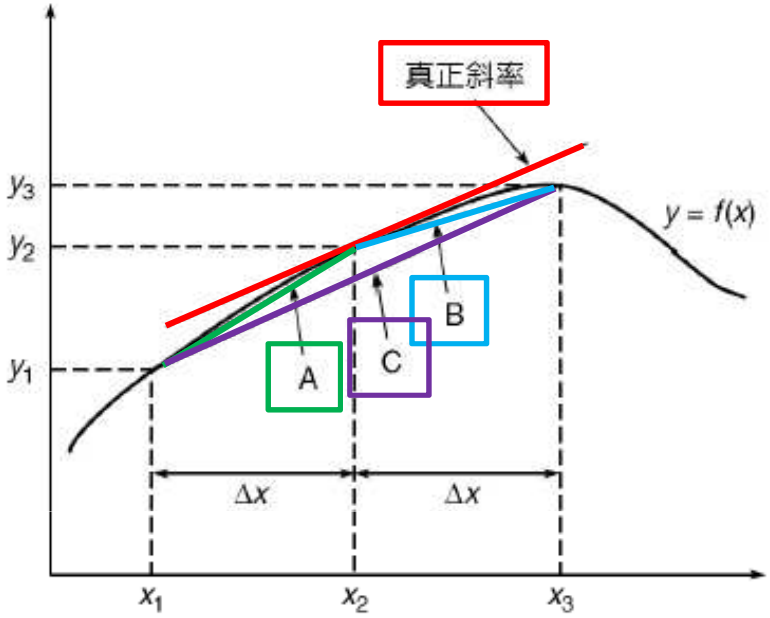
$$= 6 \times \cos(4t) \times 4$$

$$= 24\cos(4t)$$

數值微分

- 已知數據點的微分





- 在 x_2 之微分

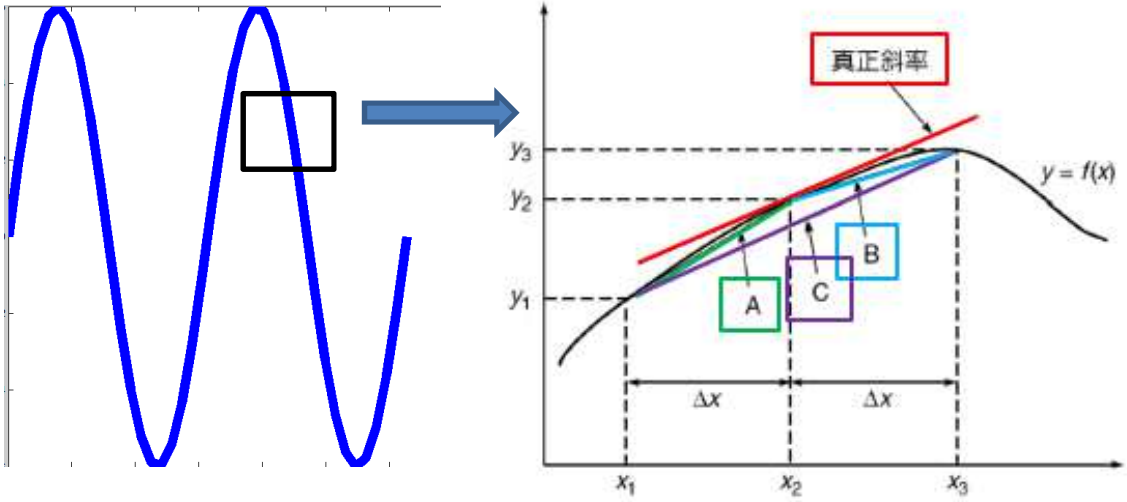
Definition of Derivative: $\dfrac{dy}{dx} = \lim_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x}$

Backward Difference: $m_A = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_2 - y_1}{\Delta x}$

Forward Difference: $m_B = \dfrac{y_3 - y_2}{x_3 - x_2} = \dfrac{y_3 - y_2}{\Delta x}$

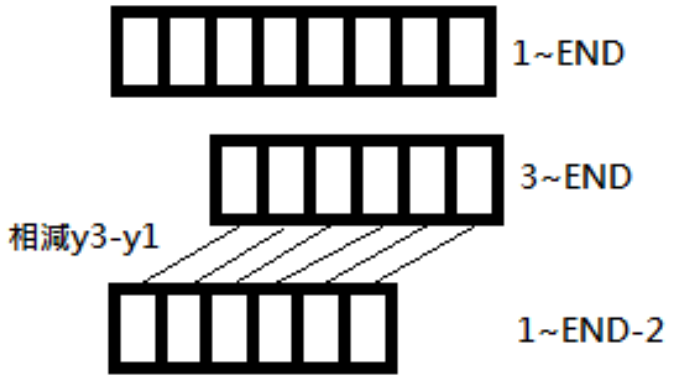
Central Difference: $m_C = \dfrac{1}{2}\left(\dfrac{y_3 - y_2}{\Delta x} + \dfrac{y_2 - y_1}{\Delta x}\right) = \dfrac{y_3 - y_1}{2\Delta x}$

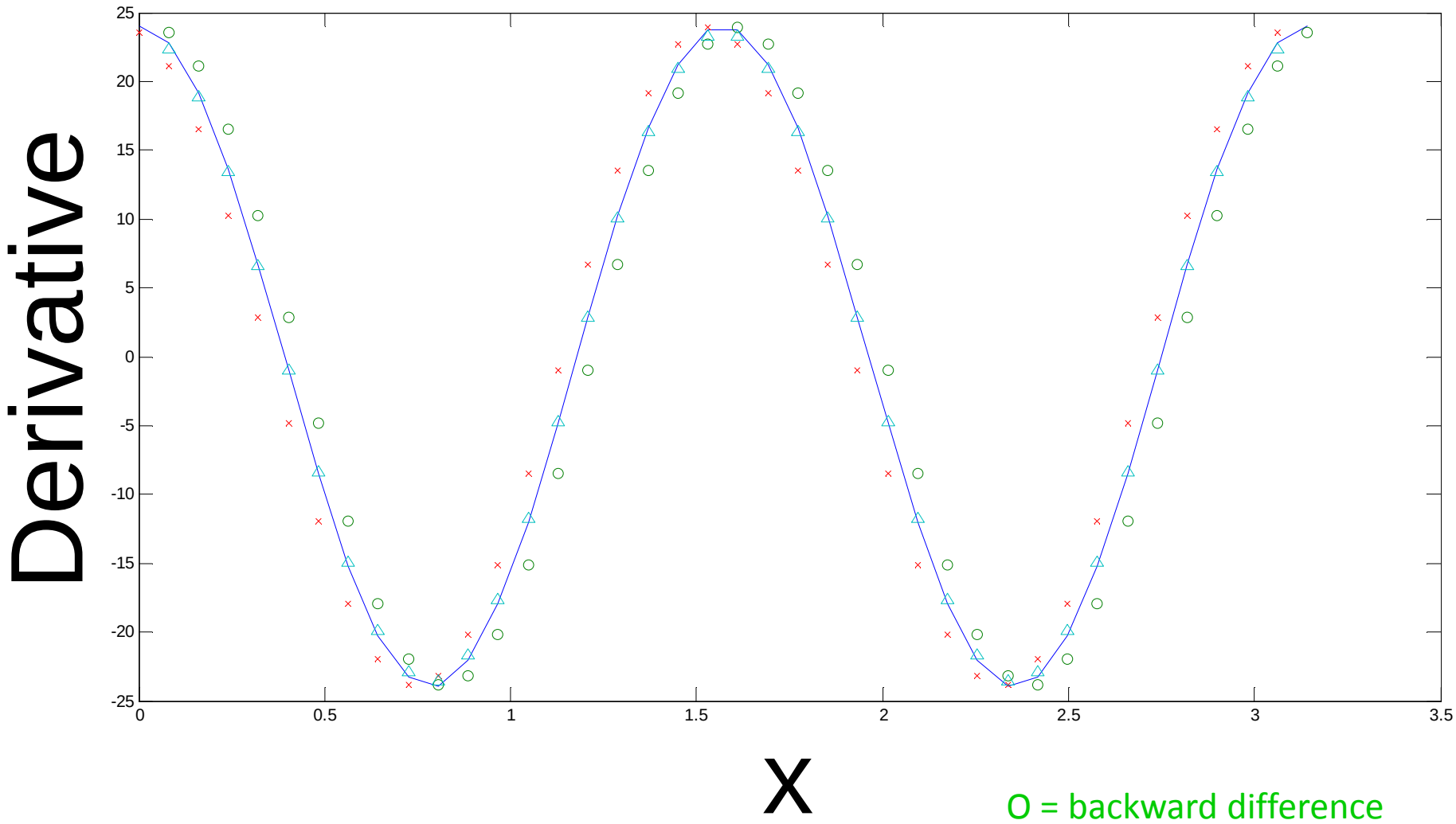
Ex: 6sin(4t):




```
>> x = linspace(0,pi,40);
>> y = 6*sin(4*x);
>> d = diff(y)./diff(x); % backward or forward difference
>> dc = (y(3:end)-y(1:end-2))./(x(3:end)-x(1:end-2)); % central
difference
>> dy = 24*cos(4*x); % 實際微分值
>> plot(x, dy, x(2:end), d,'o', x(1:end-1), d,'x', x(2:end-1), dc,'^')
>> xlabel('x','Fontsize',[60]);
>> ylabel('Derivative','Fontsize',[60])
```

$$d = \text{diff}(x)$$
$$= [x(2)-x(1), x(3)-x(2), \ldots, x(n)-x(n-1)]$$




Derivative
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
0.5
1
1.5
2
2.5
3
3.5
X
O = backward difference
X = forward difference
△= central difference
-- = 24*cos(4*x)